Matching Wealth Moments with Heterogeneous Returns

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Abstract

Recent empirical evidence of heterogeneity in the rate of return (an important feature of the wealth accumulation process) for individuals provide motivation for an analogous assumption in a standard heterogeneous agent (HA) macroeconomic model. In the infinite horizon setting, a uniform distribution of the rate of return across households is estimated such that empirical moments of wealth (net worth) measured in the Survey of Consumer Finances are matched particularly well by their model counterparts. These findings suggest that heterogeneity in parameters which determine optimal consumption-saving behavior other than the time preference factor can generate meaningful wealth inequality. Factors which explain differences in returns, on safe assets in particular, across individuals could be used to endogenize heterogeneity in the rate of return, allowing for a more robust analysis of wealth inequality using macroeconomic models.

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1 Introduction

The unequal distribution of wealth is an extensively documented phenomenon in numerous countries. Regrettably, this feature has not only endured over time but also intensified in recent years. This point is stressed in a recent article from the Institute for Policy Studies (IPS), which revealed that in 2018, the total wealth of the poorest half of Americans was eclipsed by the combined wealth of the three wealthiest men in the nation. The term "richest" denotes one's standing in Forbes magazine's list of the 400 richest individuals. Additionally, the IPS report notes that the combined wealth of the top five richest men on this list skyrocketed by a staggering 123% from March 2020 to October 2021[1].

The unequal distribution of wealth has also been a subject of considerable interest throughout history in various fields. The statistics literature, for instance, focused on linking the distribution of income to the observable skewness in wealth distribution. The economics literature went further by establishing microfoundations for wealth accumulation over the life cycle. To that end, the macroeconomics literature on inequality has seen significant growth, with the distribution of wealth among households offering insight into how the economy as a whole responds to aggregate fiscal shocks. The recent stimulus checks issued during the pandemic serve as a timely example of this phenomenon.

The macroeconomics literature has undergone significant changes in recent years, with the widespread adoption of models that abandon the traditional representative agent assumption in their analysis. As this setting will require that in equilibrium all agents hold the same level of wealth, it is not a desirable laboratory in terms of producing model objects, like the distribution of wealth, that can be compared to real world counterparts.

The first departure from the representative agent framework incorporates an exogenously determined income process that generates a distribution of income among households. One common approach to incorporating heterogeneity is to adopt Friedman 1957's description of a permanent and transitory component in the income process. To account for business cycle dynamics, one can further assume that individuals face some level of potential unemployment in each period, creating a precautionary savings motive for consumers. Given that such uncertainty cannot be fully insured against, the availability of a riskless asset that partially insures against income risk results in households choosing to hold different levels of market resources optimally.

Krusell and Smith 1998's seminal work suggests that models assuming heterogeneity in individual income perform well in matching the aggregate capital stock but poorly in matching the distribution of wealth. The next step is to assume there is some ex-ante heterogeneity among households, leading more households to optimally hold lower levels of wealth. [2] Carroll et al. 2017 adopt

[1] See Inequality.org articles data November 21, 2022: "Wealth Inequality in the United States" and "Updates: Billionaire Wealth, U.S. Job Losses and Pandemic Profiteers" (date accessed: March 27, 2023)

[2] Kaplan and Violante 2022's recent work provides a comprehensive survey of incomplete markets models with heterogeneous agents featuring (i) uninsurable idiosyncratic income risk,

this approach and assume that agents differ in their time preferences, which reflects implicit characteristics of households relevant to their lifetime wealth accumulation. The authors find that this assumption of modest heterogeneity in time preferences is sufficient to match both the shape and skewness of the empirical distribution of wealth.

The household's optimal consumption-savings problem contains additional elements that could contribute to disparities in wealth accumulation over the course of one's lifetime. It is worth noting that the time preference factor (β) is one of the key parameters that influences an individual's equilibrium target level of market resources, but it is not directly observable. Therefore, in order to estimate β, one would need to gather data through surveys or other methods that allow for the direct acquisition of information from households. On the other hand, estimating differences in the rate of return to financial assets across households is possible, as this variable *is* directly observable.

This paper aims to provide further evidence of the heterogeneous agent modelling framework's ability to match wealth moments by adding a single source of heterogeneity across households beyond the realization of ex-post shocks to their income. This is true even when there is only a single asset available to partially insure against the uncertain labor income process. I allow for households differ in the return earned on assets in a setting with rich life cycle dynamics. From there, I interpret the heterogeneity in the returns on safe assets earned by households in the context of the transmissions channel of monetary policy and its variation across the banking sector.

2 Literature Review

2.1 Collecting Data on the Distribution of Wealth

Empirical estimates of the skewness in wealth holdings over time provide valuable insights for this paper. Surveys and the imputation of wealth levels using administrative income tax data (sometimes referred to as the *capitalization method*) are the standard ways of collecting household data on the distribution of wealth for empirical analysis.

Wolff 2004 provides an early analysis of measurements of wealth by the Survey of Consumer Finances (SCF)[3] by discussing both the concentration and composition of household wealth 1980s and 1990s. The author's analysis corroborates the story of significant and growing inequality in the distribution of wealth in the U.S. Specifically, although the wealth of the average household grew in the 1990s, most of the gains in wealth and income during this period were enjoyed by the upper 20 percent of the wealth distribution, and especially the top 1 percent. While from 1983 to 2003 the top 1 percent experienced 33

(ii) a precautionary savings motive, and (iii) an endogenous wealth distribution.

[3]See Kennickell 2017a for an extensive description of the methodology for sampling the wealthist households in the SCF and Kennickell 2017b for an analysis of the performance of the SCF at measuring the wealth of the top 1 percent.

percent of the total growth in net worth (89 percent for the top 20 percent), the average wealth of the poorest 40 percent of households fell by 44 percent during this same time period and had reached roughly $2,900 by 2001.

Saez and Zucman 2014 employs the capitalization method on tax data from the Internal Revenue Service to estimate the distribution of wealth in the United States for a much longer time period of 1913 to 2012. The usefulness in the authors' approach is that they are able to decompose their measure of wealth and savings into fractiles (i.e. top 1 percent, top 10 percent, bottom 20 percent wealth shares), which allows them to analyze the evolution of wealth over time in a way that is standard in the existing literature on wealth inequality. The authors not only find that inequality in the U.S. wealth distribution is realtively high and has been growing significantly in the later periods of their dataset, but they also attribute this growth primarily to the wealthiest of households. Indeed, they cite that the wealth shares of the top .1 percent of the distribution grew from 7 percent in 1978 to 22 percent in 2012.

2.2 Explaining Inequality in the Distribution of Wealth

Benhabib and Bisin 2018 provide an insightful review of the literature on the documented skewness in the distribution of wealth. The survey begins with historical accounts of the origins of the shape of the wealth distribution, dating back as early to Pareto and Samuelson. The authors then provide the traditional theoretical explanations of this unequal distribution: (i) skewness in the (exogenous) distribution of earnings, (ii) stochastic returns to wealth and savings, and, importantly, (iii) microfoundations for the evolution of wealth resulting from the consumption and saving behavior of hosueholds[4].

Gabaix et al. 2016 define a notion for the speed of convergence to provide an explanation for observed evolution of income inequality over time, specifically in the upper tail of the distribution in the past 40 years in the United States. Notably, the authors show that, in order to match the empirical dynamics of inequality, one needs to allow for more forms of heterogeneity in the income process for households that are not incorporated in the standard consumption and saving models.[5] The first form is *type dependence* in the income growth rate distribution, which models the case in which some households have a higher average income growth rate. The second form, *scale dependence*, captures the fact that higher income levels are more susceptible to shocks to their income growth. The authors find that former does a good job at explaining this fast rise in income inequality, and the latter can generate infinitely fast transitions in inequality.[6]

[4] As explored in the next section, the emergence of heterogeneous agent models has been a significant development in investigating this issue. Bewley 1983, Aiyagari 1994, and Huggett 1993 are among the earliest examples.

[5] Note that, although this analysis is about the distribution of income, this literature notably asserts that the distirbution of wealth inherits some of its skewness from the distribution of income

[6] As we will see, these notions of "type dependence" and "scale dependence" show up in the literature on household heterogeneity and the wealth and income distributions; most

De Nardi and Fella 2017 provide another survey of the literature, more focused on the microfoundations for the distribution of wealth. Specifically, the authors note a number of possible extensions of models of household consumption and saving behavior, inspired by observable differences and the demographics of households, which lead to differences in wealth accumulation over time. Earnings and rate of return risk, ex-ante heterogeneity in preferences, medical expenses, bequest motives, and entrepreneurship are all cited as potential avenues to better explain the shape of the distirbution of wealth using the behavior of households.

2.3 Measurements of heterogeneous rates of return

The rationale behind incorporating heterogeneity in rates of return to asset holdings lies in the use of novel datasets in recent empirical research to quantify the differences in returns among individuals. Fagereng et al. 2020 document extensively the heterogeneity in realized returns using 12 years of data from Norway's administrative tax records. The authors' findings reveal substantial differences in the average returns to assets for individuals (*type dependence*), that this heterogeneity is found both within and across classes of assets with varying levels of risk, and that returns are positively correlated with wealth (*scale dependence*). Moreover, they futher demonstrate that this discovery of heterogeneous returns exhibits significant persistence over time and are positively correlated across generations. Each of these findings provide not only motivation for the assumption of ex-ante heterogeneous rates of return in the buffer-stock savings model of households, but also provide a benchmark to compare the distribution of rates of return resulting from the estimation procedure aimed at best matching the empirical distirbution of wealth, as in Carroll et al. 2017.

Bach, Calvet, and Sodini 2018 use administrative panel data on the balance sheets of Swedish residents to gauge historical and expected returns, as well as risks associated with asset holdings. Their analysis of portfolio performs supports the finding that heterogeneous returns play a considerable role in the levels and growth of top wealth shares over time.

Campbell, Ramadorai, and Ranish 2019 consider wealth held in equity accounts in India between 2002 and 2011 and find that heterogeneity in returns to investment, which can be acheived by both the inherent randomness associated with risky investment and differences in the investment strategies of investors, is a main contributor to the increase in inequality of wealth held in equity portfolios during the time period. Here, the authors attribute the scale dependence associated with the returns to equity portfolios to the finding that smaller accounts tend to be poorly diversified relative to their larger account counterparts.

Deuflhard, Georgarakos, and Inderst 2018 provide an important analysis for the heterogeneous agent, incomplete markets model with a precautionary saving motive and a *single asset* to partially insure against risk with by studying the

importantly, in the discussion on heterogeneous rates of return to wealth.

performance of households' investments in savings accounts. Not only do they find substantial type dependence in the rate of return to these safe assets, they also attribute the heterogeneity in returns to differences in financial sophistication. Notably, providing an explanation for differences in returns to investments for households is a vital step in potentially endogenizing this form of ex-ante heterogeneity among households in future research.

Altmejd, Jansson, and Karabulut 2024 is a recent work which provides causal evidence of financial education leading to significant differences in portfolio returns. Using university application data from the Swedish National Archives and data from the Swedish Income and Wealth registry, they show that indivduals marginally admitted to business or economics programs not only hold more money in stocks but earn a higher raw return on these holdings than their counterparts.

2.4 Recent HA models with heterogeneous rates of return

The paper Daminato and Pistaferri 2024 incorporates heterogeneous returns into the solution of a model of consumption-saving for households. There, they use data from the PSID to document heterogeneity in returns, which they state is comparable to that found in the Norwegian registry data used by Fagereng et al. 2020.

Benhabib, Bisin, and Luo 2019 proposes an overlapping generations model that incorporates intergenerational wealth transfers. There, agents face uncertainty regarding both labor and capital income. Benhabib, Bisin, and Luo 2017 undertake a similar exercise, where household preferences for bequests to the next generation are more explicitly defined. Both papers conclude that the distribution of earnings and differences in rates of savings and bequests are crucial in matching the characteristics of the observed wealth distribution's tail ends.

Guler, Kuruscu, and Robinson 2022 develop a life-cycle model that provides a comprehensive description of households' optimal decision-making to endogenize heterogeneity in the rate of return, as they consider optimal choices regarding housing and mortgage decisions. These modeling choices enable the authors to investigate the effects of aggregate fiscal shocks, including one-time stimulus payments and mortgage debt relief programs.

Menzio and Spinella 2025 introduce a model of the financial market with search frictions into the standard macroeconomic setting of an infinite-horizon decision problem for households and firms. A distribution of returns across households arises endogenously in their model, and they use the empirical findings from Fagereng et al. 2020 regarding the distribution of returns to net worth as notable targeted moments.

2.5 My contributions

After taking stock of the research focused on returns heterogeneity and the distribution of wealth, my paper is different from the literature in at least two ways. The first notable difference is that, my paper models the uncertainty in labor income as a random walk, as opposed to an AR(1) process. The key implication of this modeling difference is that, the AR(1) specification leads to less uncertainty in earnings over the life cycle from the perspective of a household at the start of the horizon for which they are choosing an optimal consumption path. This will lead to less accumulation of wealth over the life cycle. In this way, the permanent income specification that I am using will attempt to describe *as much* of the dispersion of wealth across households as possible with labor income uncertainty. The remaining dispersion in wealth across households which cannot be explained by differences in earnings will be attributed to returns heterogeniety, ultimately leading to more modest estimates of differences in returns acrosss households.

The second way in which may paper is different from the existing literature is that the life cycle version of my model is much richer in its calibration of earnings and mortality rates. Specifically, I use an earnings profile provided by Cagetti 2003, which distinguishes mean earnings not only by age but by education cohort. Furthermore, I use age-education dependent mortality rates provided by Brown, Liebman, and Pollet 2007, which is uncommon in this literature. Essentially, I distinguish between households demographically by more than just age by incorporating education levels. Doing so does provide another avenue to explain dispersion in wealth holdings across households. However, it is a limited role and some further ex-ante heterogeneity among households, such as time preferences or the rate of return, is still needed to match wealth moments precisely.

3 Model

3.1 Defining the stochastic income process

Each household's income (y_t) during a given period depends on three main factors. The first factor is the aggregate wage rate (W_t) that all households in the economy face. The second factor is the permanent income component (p_t), which represents an agent's present discounted value of human wealth. Lastly, the transitory shock component (ξ_t) reflects the potential risks that households may face in receiving their income payment during that period. Thus, household income can be expressed as the following:

$$y_t = p_t \xi_t W_t.$$

The level of permanent income for each household is subject to a stochastic process. In line with Friedman 1957's description of the labor income process, we assume that this process follows a geometric random walk, which can be expressed as:

$$p_t = p_{t-1} \psi_t,$$

The white noise permanent shock to income with a mean of one is represented by ψ_t, which is a significant component of household income. The probability of receiving income during a given period is determined by the transitory component, which is modeled to reflect the potential risks associated with becoming unemployed. Specifically, if the probability of becoming unemployed is $\mho$, the agent will receive unemployment insurance payments of $\mu > 0$. On the other hand, if the agent is employed, which occurs with a probability of $1 - \mho$, the model allows for tax payments τ_t to be collected as insurance for periods of unemployment. The transitory component is then represented as:

$$\xi_t = \begin{cases} \mu & \text{with probability } \mho, \\ (1-\tau_t) l \theta_t & \text{with probability } 1-\mho, \end{cases}$$

where l is the time worked per agent and the parameter θ captures the white noise component of the transitory shock.

3.2 Decision problem for households

This paragraph presents the baseline version of the household's optimization problem for consumption-savings decisions, assuming no ex-ante heterogeneity. In this case, each household aims to maximize its expected discounted utility of consumption $u(c) = \frac{c^{1-\rho}}{1-\rho}$ by solving the following:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} (\cancel{D}\beta)^n u(c_{t+n}).$$

It's worth noting that the setting described here follows a perpetual youth model of buffer stock savings, similar to the seminal work of Krusell and Smith 1998. To solve this problem, we use the bellman equation, which means that the sequence of consumption functions $\{c_{t+n}\}_{n=0}^{\infty}$ associated with a household's optimal choice over a lifetime must satisfy[7]

$$\begin{aligned} v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \cancel{D} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v(m_{t+1})] \\ &\text{s.t.} \\ a_t &= m_t - c_t(m_t), \\ k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}}, \\ m_{t+1} &= (1+r_t)k_{t+1} + \xi_{t+1}, \\ a_t &\geq 0. \end{aligned}$$

[7]Here, each of the relevant variables have been normalized by the level of permanent income ($c_t = \frac{C_t}{p_t}$, and so on). This is the standard state-space reduction of the problem for numerical tractibility.

4 Results

4.0.1 The model with no returns heterogeneity

To solve and simulate the model, I follow the calibration scheme captured in table 1.

Description	Parameter	Value	Source
Time discount factor	β	0.99^4	Den Haan, Judd, and Juillard 2010
CRRA	ρ	1	Den Haan, Judd, and Juillard 2010
Capital share	α	0.36	Den Haan, Judd, and Juillard 2010
Depreciation rate	δ	0.025	Den Haan, Judd, and Juillard 2010
Time worked per employee	ℓ	$1/.09$	Den Haan, Judd, and Juillard 2010
Wage rate	W	2.37	Den Haan, Judd, and Juillard 2010
Unempl. insurance payment	μ	0.15	Den Haan, Judd, and Juillard 2010
Probability of survival	$\not{D}$	$(1 - 0.00625)^4$	Yields 40-year working life
Std. dev of $\log \theta_{t,i}$	σ_θ^2	$0.010 \text{ x } 4 \text{ x } \sqrt{4}$	Carroll 1992, Carroll, Slacalek, and Tokuoka 2015
Std. dev of $\log \psi_{t,i}$	σ_ψ^2	$0.010 \text{ x } 4/11 \text{ x } \sqrt{4}$	Carroll 1992, Debacker et al. 2013, Carroll, Slacalek, and Tokuoka 2015
Unemployment rate	$\mho$	0.07	Mean in Den Haan, Judd, and Juillard 2010

Table 1: Parameter values (annual frequency) for the perpetual youth model.

The solution of the model with no heterogeneity in returns (the R-point model) is the one which finds the value for the rate of return R which minimizes the distance between the simulated and empirical wealth shares at the 20th, 40th, 60th, and 80th percentiles. The empirical targets are computed using the 2004 SCF data on household wealth. The estimation procedure finds this optimal value to be $R = 1.0602$.

4.0.2 Incorporating heterogeneous returns

As noted above, recent studies by Fagereng et al. 2020 and Bach, Calvet, and Sodini 2018 have not only estimated the rate of return on asset holdings but have also uncovered significant heterogeneity across households. With this in mind, the next estimation (the R-dist model) assumes the existence of multiple types of agents, each earning a distinct rate of return on their assets.

I follow closely the procedure outlined by Carroll et al. 2017. Specifically, I assume that different types of households have a time preference factor drawn from a uniform distribution on the interval $(\grave{R} - \nabla, \grave{R} + \nabla)$, where ∇ represents the level of dispersion. Afterward, the model is simulated to estimate the values of both $\grave{R}$ and ∇ so that the model matches the inequality in the wealth distribution. To achieve this, the following minimization problem is solved:

$$\{\grave{R}, \nabla\} = \arg\min_{R,\nabla} \left(\sum_{i=20,40,60,80} (w_i(R, \nabla) - \omega_i)^2 \right)^{\frac{1}{2}}$$

subject to the constraint that the aggregate capital-to-output ratio in this model matches the calibrated value 5. This is towards the upper bound on plausibly calibrated values for the capital-to-output ratio, as much of the literature chooses values between 2 and 3.

Note that w_i and ω_i give the porportion of total aggregate net worth held by the top i percent in the model and in the data, respectively.

The estimation procedure finds this optimal values of $R = 1.0212$ and $\nabla = 0.06728$. These parameter values pin down the estimated uniform distribution. In the model, I've chosen to discretize that distribution to 7 chosen points. The performance of the estimation of both the R-point and R-dist models, measured by their ability to match the SCF data, is compared in figure 1.

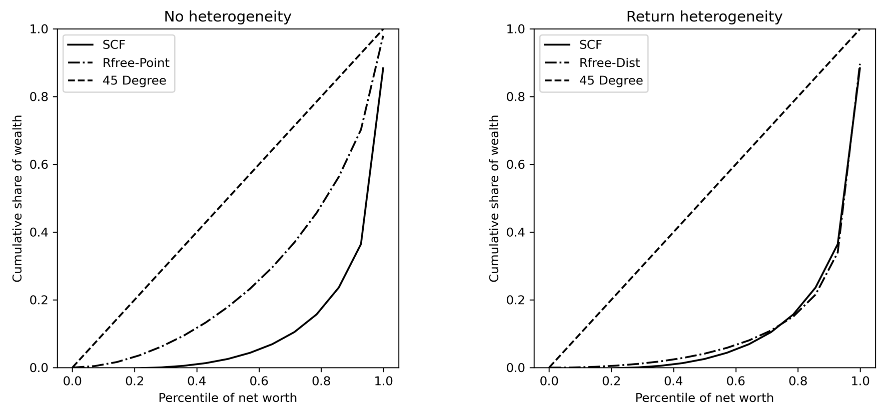

Figure 1: Comparison of R-Point and R-Dist Models.

4.1 Incorporating life cycle dynamics into the model

More realistic assumptions regarding the age and education level of households can have important implications for the income and mortality process of households. Here, I extend the model to incorporate these life cycle dynamics.

Households enter the economy at time t aged 24 years old and are endowed with an education level $e \in \{D, HS, C\}$, and initial permanent income level $\mathbf{p}_0$, and a capital stock k_0. The life cycle version of household income is given by:

$$y_t = \xi_t \mathbf{p}_t = (1-\tau)\theta_t \mathbf{p}_t,$$

where $\mathbf{p}_t = \psi_t \bar{\psi}_{es} \mathbf{p}_{t-1}$ and $\bar{\psi}_{es}$ captures the age-education-specific average growth factor. Households that have lived for s periods have permanent shocks drawn from a lognormal distribution with mean 1 and variance $\sigma^2_{\psi s}$ and transitory shocks drawn from a lognormal distribution with mean $\frac{1}{\not\mho}$ and variance $\sigma^2_{\theta s}$ with probability $\not\mho = (1-\mho)$ and μ with probability $\mho$.

The normalized version of the age-education-specific consumption-saving problem for households is given by

$$
\begin{aligned}
v_{es}(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta \not{D}_{es} \mathbb{E}_t[\psi_{t+1}^{1-\rho} v_{es+1}(m_{t+1})] \\
&\text{s.t.} \\
a_t &= m_t - c_t, \\
k_{t+1} &= \frac{a_t}{\psi_{t+1}}, \\
m_{t+1} &= (1 + r_t)k_{t+1} + \xi_{t+1}, \\
a_t &\geq 0.
\end{aligned}
$$

The additional parameters necessary to calibrate the life cycle version of the model are given in table 2. The age-education dependent mean income levels come from Cagetti 2003. The permanent and transitory shock variances come from Sabelhaus and Song 2010. The age-education dependent mortality rates come from Brown, Liebman, and Pollet 2007.

Description	Parameter	Value
Population growth rate	N	0.0025
Technological growth rate	Γ	0.0037
Rate of high school dropouts	θ_D	0.11
Rate of high school graduates	θ_{HS}	0.55
Rate of college graduates	θ_C	0.34
Labor income tax rate	τ	0.0942

Table 2: Parameter values (annual frequency) for the lifecycle model.

The estimation procedure finds this optimal value to be $R = 1.0414$ for the R-point model in this setting. The estimation procedure for the R-dist model in the life cycle setting finds optimal values of $R = 1.0343$ and $\nabla = 0.03557$. Notice the improved performance of the estimation in matching the data displayed in figure 2.

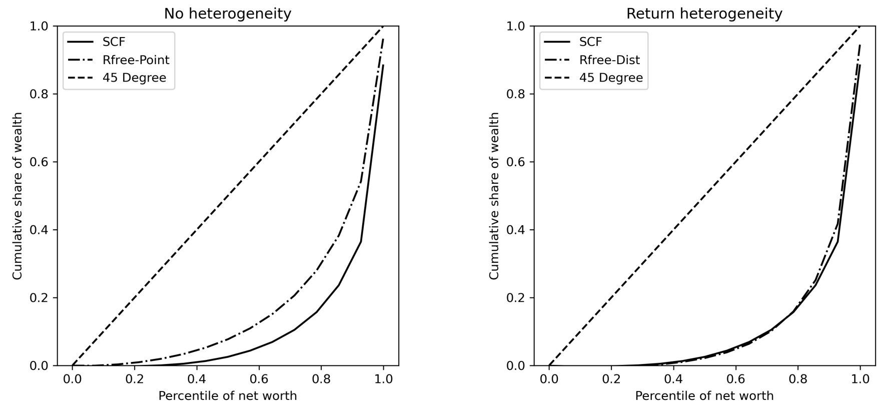

Figure 2: Comparison of R-Point and R-Dist Models in the Life-Cycle Setting.

4.2 Untargeted moments

It is well-documented in the literature on heterogeneous agent modeling that, adding a source of (ex-ante) heterogeneous beyond the addition of labor income uncertainty to the representative agent framework will allow the simulated distribution of wealth to match moments of the empirical wealth distribution particularly well. So, although it is useful to see that the model with heterogeneous returns does a good job of matching the given lorenz targets, we need another way to assess the model's performance.

For this reason, I include age-dependent wealth moments from the same wave of the SCF to serve as untargeted moments 3. These can be found in the following table.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0723	-0.0657	-0.0266	0.1099
30-40	-0.008	0.0054	0.057	0.1813
40-50	-0.0001	0.0187	0.0776	0.2178
50-60	0.0018	0.0215	0.0766	0.2126
60-70	0.0011	0.0188	0.0726	0.2081

Figure 3: Empirical Lorenz Curve Targets from the 2004 SCF.

The next two tables present the simulated version of the untargeted moments for the model without heterogeneity 4, and then with heterogeneity 5. As you can see from the tables below, the age-dependent Lorenz targets that arise from the model again fit the data much better when returns heterogeneity is present versus when it is absent.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0046	0.0368	0.1032	0.243
30-40	0.0068	0.0487	0.1341	0.3062
40-50	0.0161	0.0675	0.1666	0.3573
50-60	0.0234	0.0809	0.1858	0.3772
60-70	0.0226	0.0776	0.1792	0.3671

Figure 4: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0024	0.0242	0.0859	0.2242
30-40	-0.0124	0.0064	0.0662	0.2221
40-50	-0.0088	0.0046	0.0545	0.2077
50-60	-0.0006	0.0157	0.069	0.2234
60-70	0.0038	0.0239	0.0809	0.2341

Figure 5: Simulated Untargeted Moments with Heterogeneity (R-dist).

5 Wealth v.s. Capital income tax

It is well known in the literature that the interesting setting to compare the effects of a tax on wealth versus one on capital income is when heterogeneous returns are present. Guvenen et al. 2023 documents that, when individuals earn different returns on their assets, a capital income tax will place a larger burden on those more efficient with their capital. On the other hand, the wealth tax will shift the burden towards those who are unproductive with their capital. This redistributive effect of the wealth tax leads to higher aggregate productivity, output, and ultimately larger welfare gains than the capital income tax. Following a similar line of reasoning, in this setting we can consider the quantitative effects of applying each of the tax schemes on distribution of wealth when heterogeneous returns are present.

Suppose we begin in an economy where the distribution of returns is the one needed to best match wealth moments measured in the data. The question then is, what happens to the distribution of wealth if a revenue equivalent tax rate is applied to every household? It is clear that a capital income tax will decrease wealth inequality, since households that earn any capital income will see a tax but those with zero or negative returns will see a capital tax of zero. This will lead to less wealthy households holding a higher share of the aggregate wealth than when the capital tax is not present. The effects of the wealth tax are less obvious, since all households hold some wealth and thus will be taxed accordingly.

In this setting, the wealth tax enters the household budget constraint in the following way:

$$m_{t+1} \;=\; \big(1 + r_t - \tau_w\big)\, k_{t+1} + \xi_{t+1}.$$

. Similarly, for the capital income tax, we have

$$m_{t+1} \;=\; \big(1 + (1 - \tau_{ci})\, r_t\big)\, k_{t+1} + \xi_{t+1}.$$

. I compute aggregate income as GDP in this setting, and then find the wealth tax rate and the capital tax rate which would raise tax revenues which are equal to 1% of GDP. I do this for the estimated distribution of returns both for the infinite horizon and the life cycle setting.

From there, I apply the tax schemes to each households and find the resulting wealth distributions for both cases. The wealth moments can be easily compared before and after each of the policies were implemented. Below are the results of applying each of the tax schemes in the infinite horizon 3 version of the model.

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	.87%	4.1%	11.1%	25.4%
Wealth $\tau_w = .56\%$	1%	5.1%	14%	31.4%
Capital income $\tau_{ci} = 10.3\%$	1.2%	5.7%	15.4%	34.1%

Table 3: Tax policies in the infinite horizon setting.

Applying the tax schemes in both cases increases the share of the wealth held at each of the chosen percentiles. This makes sense, as such a tax should redistribute extreme levels of wealth towards lower rungs of the distribution. However, the effect is mininal in the case of the flat wealth tax, and especially pronounced for the capital income tax. This is in line with wha tthe literature regarding the effects of these two policies when heterogeneous returns are present: if the capital income tax places a burden on the *most productive* households regarding use of capital, then it should lead to less wealth inequality since those households contribute most to the skewness in the distribution.

Next, I apply the tax schemes in the life cycle 4 version of the model. A similar result arises here, where the capital income tax has a larger effect on reducing wealth inequality than the wealth tax which raises the same level of tax revenue does. From the table, the effects in the life-cycle model are less pronounced than they are for the infinite horizon case.

	Lorenz points			
Tax scheme	20%	40%	60%	80%
None	.7%	3.2%	9.4%	25.2%
Wealth $\tau_w = .36\%$	.73%	3.4%	9.7%	25.6%
Capital income $\tau_{ci} = 7.1\%$	.79%	3.6%	10.3%	26.6%

Table 4: Tax policies in the life cycle setting.

6 Mechanism for returns heterogeneity

There have been a number of potential explanations proposed regarding the persistent component of returns heterogeneity. Among the more common ones is the idea that there are some business owners in the economy, and variability among their entrepreneurial talent leads to even more heterogeneity in labor income than the assumptions about labor income uncertainty that the standard HA models use. Cagetti and De Nardi 2006 and Cagetti and De Nardi 2009 are notable exmaples of explicitly modeling entrepreneurial talent as a compoenent of the consumption-saving problem for households and assesing the ability of such models in matching empirical wealth moments.

Although differences in entreprenuerial talent modeled as variability in labor market productivity allows for the model to better match wealth moments at the upper tail of the distribution, it is an unsavory explanation for the mechanism I have in mind in this paper. Namely, this modeling choice will result in households (firms) with high levels of wealth (capital) earning lower rates of return, and vice versa for households (firms) with low levels of wealth (capital). As mentioned before, Fagereng et al. 2020 documents scale dependece regarding wealth heterogeneity: that returns, as well as the idiosyncratic, persistent component of returns, are positively correlated with wealth.

Another explanation from the literature which is closer to, but still not exactly the same as, the mechanism in this paper which allow two households with the same level of assets to earn a different return on them is financial literacy or sophistication. Lusardi and Mitchell 2014 offer a survey on models which explicitly allow for households to make a costly decision to build up a stock of financial literacy, in turn allowing them access to an investment technology offering a higher average return. Lusardi, Michaud, and Mitchell 2017 show that allowing for such endogeneous financial accumulation in the standard consumption-saving framework again allows the model to match wealth moments particularly well, this time through the returns channel (as opposed to the labor income channel associated with entrepreneurial talent).

As I am particularly interested in a setting with a single, safe asset available, I investigate another cause for heterogeneity in the rate of return by following literature which documents substantial heterogeneity in the banking sector, specifically on rates offered to depositors.

6.1 Deposit rates and the sensitity of deposit levels to changes in the market interest rate

As mentioned before, Deuflhard, Georgarakos, and Inderst 2018 and others have shown that there can be substational heterogeniety in rates of return earned by depositors. The key step in incorporating this finding in the standard consumption-saving framework is to identify potential sources of this heterogeniety in banking.

Consider the balance sheet of banks within the financial sector and the problem they face in optimally choosing the rate to offer on deposits. In a simple setting, they accept deposits at an offered deposit rate, and then hold reserves within the central banking system which earn the market interest rate. This discrepancy between the offered deposit rate and the market interest rate leaves room for banks to earn profit.

It is well documented in the U.S. empirically that the level of deposits at a given bank will change due to exogenous changes in the Fed funds rate. Drechsler, Savov, and Schnabl 2017 propose a clear transmissions channel for monetary policy: changes in the Fed funds rate may lead banks to widen the interest spread they charge on deposits, which causes deposits to flee the bank. The authors find a strong, negative relationship between changes in the federal funds rate and the growth rate of deposits. Furthermore, they also note that a 100 basis point increase in the Fed funds rate leads to a higher deposit outflow in bank branches in more concentrated markets relative to those in less concentrated markets.[8]

With this in mind, variation in the strength of this transmission channel across banks can be viewed as a potential source of heterogeneity in the banking sector, as certain market characteristics would make the level of deposits offered by some banks less sensitive to changes in federal funds rate than other banks. For example, Sarkisyan and Viratyosin 2021 make a distinction between *local* and *globally integrated* banks, and show that "global banks lose much more deposits relative to local banks in response to unexpected changed in the federal funds rate". Adrien d'Avernas et al. 2024 show a similar finding regarding heterogeneity in deposit rates, but for larger vs smaller banks.

With this in mind, the next step is to extend the standard HA model describing household consumption-saving decisions by explicitly modeling a banking sector. In this setting, banks will each solve a similar profit-maximization problem regarding accepting deposits at an offered deposit rate and holding reserves which earn the market interest rate. The key distinction between banks in the model will be how sensitive the level of deposits are to changes in the market

[8] Branches in "more concentrated markets" refers to banks which operate in local deposit markets where a few banks hold large market shares.

interest rate, which is in line with the mentioned empirical evidence. From here, it will be clear how heterogeneity in returns may arise for households which are otherwise the same.

6.2 Model of heterogeneous deposit rates

Here I present a small, open economy with banks and households as the optimizing agents in the model. This will be a partial equilibrium analysis since the world interest rate is being taken as given. That said, I present a simple framework for describing the optimal behavior for banks setting deposit rates. By assuming that there is a cobb-douglas aggregate production function, I will find the marginal product of capital (less depreciation) that is consistent with the capital to output ratio from the model which matches its empirical counterpart. This effective interest rate will be considered the world or "market" interest rate and will be used along with the estimated distribution of heterogeneous returns to back out esetimated values for elasticites of foreign deposits to the deposit rates for each of the banks in the model.

6.2.1 Assumptions regarding the banking sector

There are a continuum of banks, identical in all respects other than the elasticity of the level of deposits to changes in the market interest rate.

The model is static in that, I assume that the bank chooses a deposit rate to offer to its clientele base at the start of the time horizon. Therefore, this decision depends soley on the market interest rate and the bank's given elasticity. Additionally, a given bank cannot take actions to increase the number of depositors at their given institution [9].

Lastly, in this simple version of the model, I assume that households do not endogenously choose which banks to do business with. Clearly, this would lead us towards the literature on costly human capital acquisition and financial literacy. Instead, I assume that at the outset banks are assigned to a household at birth with some probability. The household is "stuck" with this bank assignment until death. In this way, the banking sector merely replaces the assignment of idiosyncratic rates of returns over the time horizon in the standard model.

6.2.2 Decision problem for banks accepting deposits

The sole distinction between banks in this model is the sensitivity of their level of deposits to changes in the market interest rate, which we will index by ε_i. This will be the source of returns heterogeneity in the model. To see this, I follow a similar, but simplified description of the decision problem for banks which can be found in Paul and Ulate 2024.

Let R^m be the market rate of return, R^d be the rate of return offered on deposits by a bank, and $S(R^d, R^m)$ be the level of deposits held at a given bank.

[9] For example, compare a bank in a suburb area of Montana (local) versus a bank near downtown Houston, Texas (globally integrated).

Banks solve:

$$\max(R^m - R^d) \cdot S(R^d, R^m)$$

subject to:

$$S(R^d, R^m) = A\left(\frac{R^d}{R^m}\right)^{\varepsilon}$$

Importantly, the first order condition implies that the optimal deposit rate for the i-th bank is given by

$$R_i^d = \frac{\varepsilon_i}{1+\varepsilon_i} R^m$$

. This is crucial for the model in that, so long as we calibrate the model for a particular value of R^m, estimating a uniform distribution of returns using the simulated method of moments will imply a corresponding distribution of elasticities (the one that minimizes the distance between simulated and lorenz wealth moments). In this way, the 7 discretized points capture 7 different deposit rates offered, which result in varying elasticities among 7 different bank types in the model. From the expression above, we see that banks with higher values of ε_i must set R_i^d closer to R^m.

6.3 The implied distribution of bank heterogeneity

I've estimated the distribution of returns which matches the wealth moments. As mentioned earlier, I can use this and the assumptions regarding the bank's decision problem to back out an implied distribution of ε. This will describe how the banks differ in the sensitivity of their deposit levels to changes in the market interest rate. Both in our simplified setting, and in the transmission channel empirically documented by Drechsler, Savov, and Schnabl 2017, differences in these sensitivities ultimately leads to differences in the deposit rates offered across banks.

First, I assume that the aggregate production function is Cobb-douglas, so that the marginal product of capital can be written as $\alpha\frac{Y}{K}$. With the calibrated values $\delta = .025$, $\alpha = .36$, and the capital to output ratio 3, this setting has an effective interest rate of $R^m = 1.095$ which can be used as the market interest rate.

Since the model with heterogeneity (i.e. the R-dist model) has 7 estimated points for the uniform distribution, the implied, estimated points for ε can be uniquely pineed down by the expression

$$\varepsilon_i = \frac{R_i^d}{R^m - R_i^d}.$$

For the infinte horizon version of the model which matches 2004 SCF data on net worth, the 7 estimated points describing heterogeneous returns are `[0.9635, 0.9828, 1.0012, 1.0212, 1.0404, 1.0596, 1.0789]`. The corresponding 7 implied elasticities are given by `[7.3288, 8.7552, 10.7712, 13.8374, 19.0636, 29.9737, 66.8914]`.

For the life cycle version of the model which matches 2004 SCF data on net worth, the 7 estimated points describing heterogeneous returns are `[0.9755, 0.9913, 1.0072, 1.0230, 1.0388, 1.0546, 1.0705]`. The corresponding 7 implied elasticities are given by `[8.1649, 9.5642, 11.4677, 14.2079, 18.4920, 26.1362, 43.6448]`.

6.3.1 Interpreting the implied distribution of elasticities

Another way to assess the model's performance is to return to the literature on bank deposit sensitivities to changes in the federal funds rate. As we will see, the implied distribution of elasticities from my estimation method can be directly compared to those empirical estimatese, and how well they match can be used to assess my model. Additionally, I include wealth shares by age cohort as an additional set of untargeted moments for a similar assessment.

The usefulness in choosing a functional form for the level of deposits at a given bank as $S(\cdot) = A\left(\frac{R^d}{R^m}\right)^{\varepsilon}$ is that the parameter ε has a clear interpretation as the elasticity of deposits to changes in the market interest rate. It can be shown that:

$$-\varepsilon = \frac{\partial \ln S(\cdot)}{\partial \ln R^m}.$$

So, the elasticity parameter tells us how a percentage change in the market interest rate changes the level of deposit, in percent terms. This allows us to directly compare the implied elasticities following the SMM procedure to the emepirical evidence on the transmission channel described by Drechsler, Savov, and Schnabl 2017 regarding the relationship between the Fed funds rate and the level of deposits at banks. For example, Genay and Halcomb 2004 finds that a 1% change in the Fed funds rate leads to about a 3% to 4% change in the level of deposits, depending on the size of the bank.

The amount of returns heterogeneity required to match wealth inequality using only safe assets (i.e. bank deposits) will lead to vastly overstated elasticities for the resulting banking sector. This is not surprising. If bank wish to attract depositors in the face of an increasing Fed funds rate, they will need to offer a higher rate on deposits, regardless of the size of the bank. This suggests that there will be less variation in the optimal deposit rates offered across the banking sector. The banks which do not offer competitive deposit rates will likely find that their depositors switch to other safe investment technologies like money market funds. Since my model doesn't not match any moments regarding the number of banks in the economy, nor does it model returns heterogeneity by allowing for the choice between safe assets, it isn't too surprising that the elasticity of deposits to changes in the market interest rate is not well matched in this setting. That said, the ability of the model to back out a distribution of elasticities under the given assumptions is still useful.

7 Conclusion

I find the ex-ante heterogeneity in rates of return needed to match wealth moments, which is a common practice in heterogeneous agent macroeconomics. The model sits well with other deviations from the representative agent framework in that it does a good job of producing a simulated distribution of wealth with significant skewness when compared to its empirical counterpart.

This paper is slightly different from the literature which intersects HA models and evidence of a persistent component to returns in that I focus less on common explanations for the latter (like entrepreneurial talent and financial sophistication). Instead, I focus more on heterogeneity in the banking sector regarding offered deposit rates. I incorporate that literature in the standard HA framework with a simple, but realistic story in that many households may be "stuck" with the bank in or around their neighborhood. That bank has complex financial decisions to make, which ultimately trickles down to the household through the channel of verying deposit rates offered.

Although I leave out the possibility of households switching to one bank or another, this story has a similar essence to the financial literacy story when attempting to explain how returns may be heterogeneous across individuals. However, it leaves out the risk associated with portfolio choice. This is a nice feature, since (i) untangling how much of the persistent component of returns comes from risk preferences and from financial sophistication is not so straightforward and (ii) there is significant heterogeneity in returns even when individuals hold no risky assets.

As an aside, this model is still a partial equilibrium analysis. The market interest rate is being taken as given. It is not determined by some market clearing condition.

I view my model as the simplest implementation of a potential source of heterogeneity. With that in mind, in the simulation of the model and the resulting SMM estimation, I do not add banks as an agent type. Thus, the bank is not responding in every period to the level of deposits they receive after they set the optimal deposit rate based on the demand for foreign deposits that they face. This also means that I avoid choosing a particular scheme of allocating agents in the model to a particular bank. In this way, there are 7 types of banks just as there are 7 types of returns that an agent may receive.

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Appendix

Here I include the results of the SMM procedure when matching wealth data from different waves of the SCF survey. The general takeaway is that the fit of the model is robust to using different years of wealth data. One notable finding is that, for these later waves of the survey, the lowest estimated point is smaller and the highest estimated point is higher, corresponding with more heterogeneity in returns.

A Results from 2007 wealth data

A.1 Estimated distribution of returns

This table provides the results from the four versions of the model (either infinite horizon or life cycle with and without heterogeneity) as described my the mean and standard deviation of the uniform distirbution which makes the simulated wealth moments closest to the wealth moments measured in the 2007 survey.

	Mean	St. Dev
PY-Point	1.060	0.0
PY-Dist	1.020	0.012
LC-Point	1.043	0.0
LC-Dist	.999	0.016

A.2 Implied elasticities

Here are the results for the seven estimated points for the discretized uniform distribution and the 7 implied elasticies for each of the variations of the model matching 2007 SCF data.

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.960	7.120	0.916	5.126
0.980	8.518	0.944	6.253
0.999	10.498	0.972	7.889
1.020	13.517	0.999	10.479
1.039	18.688	1.027	15.197
1.059	29.578	1.055	26.499
1.079	67.432	1.083	90.016

A.3 Untargeted moments

These three tables present the wealth moments by age cohort for the 2007 wave of the SCF 6 , and the simulated version of these untargeted moments for the life cycle version of the model without heterogeneity 7 and then with heterogeneity 8.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.0513	-0.0462	-0.0207	0.0958
30-40	-0.0099	0.0023	0.0561	0.2088
40-50	-0.0007	0.0162	0.0742	0.2093
50-60	0.0018	0.0212	0.0774	0.1983
60-70	0.0025	0.0244	0.0737	0.1917

Figure 6: Empirical Lorenz Curve Targets from the 2007 SCF.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0448	0.1342	0.277	0.525
30-40	0.0418	0.1419	0.2975	0.5316
40-50	0.0461	0.1408	0.2863	0.5104
50-60	0.0503	0.1446	0.287	0.506
60-70	0.0465	0.1349	0.2701	0.4832

Figure 7: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0223	0.0941	0.2188	0.4437
30-40	0.0097	0.0509	0.1401	0.3394
40-50	0.0055	0.0286	0.0859	0.2613
50-60	0.0042	0.0209	0.0673	0.2347
60-70	0.0037	0.0188	0.064	0.2245

Figure 8: Simulated Untargeted Moments with Heterogeneity (R-dist).

B Results from 2010 wealth data

B.1 Estimated distribution of returns

Here are the results from the four versions of the model which makes the simulated wealth moments closest to the wealth moments measured in the 2010 survey.

	Mean	St. Dev
PY-Point	1.060	0.0
PY-Dist	1.002	0.015
LC-Point	1.042	0.0
LC-Dist	.981	0.020

B.2 Implied elasticities

Here are the results for the seven estimated points for the discretized uniform distribution and the 7 implied elasticies for each of the variations of the model matching 2010 SCF data.

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.923	5.396	0.876	3.995
0.950	6.540	0.911	4.947
0.976	8.184	0.946	6.347
1.002	10.743	0.981	8.608
1.028	15.280	1.016	12.882
1.054	25.531	1.051	24.002
1.080	70.639	1.086	124.683

B.3 Untargeted moments

These three tables present the wealth moments by age cohort for the 2010 wave of the SCF 9 , and the simulated version of these untargeted moments for the life cycle version of the model without heterogeneity 10 and then with heterogeneity 11.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.1122	-0.1044	-0.0603	0.1096
30-40	-0.0552	-0.0469	-0.007	0.1131
40-50	-0.0099	-0.0	0.0376	0.1402
50-60	-0.0039	0.0074	0.0496	0.1624
60-70	0.0002	0.0178	0.0651	0.1861

Figure 9: Empirical Lorenz Curve Targets from the 2010 SCF.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0446	0.1342	0.2769	0.5248
30-40	0.0415	0.1414	0.2968	0.5309
40-50	0.0458	0.1401	0.2853	0.5094
50-60	0.0501	0.1441	0.2864	0.5053
60-70	0.0464	0.1346	0.2697	0.4828

Figure 10: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0166	0.0814	0.1979	0.4175
30-40	0.0062	0.0392	0.1154	0.298
40-50	0.0035	0.021	0.066	0.2154
50-60	0.0027	0.0145	0.0474	0.1849
60-70	0.0023	0.0121	0.043	0.1738

Figure 11: Simulated Untargeted Moments with Heterogeneity (R-dist).

C Results from 2013 wealth data

C.1 Estimated distribution of returns

Here are the results from the four versions of the model which makes the simulated wealth moments closest to the wealth moments measured in the 2013 survey.

	Mean	St. Dev
PY-Point	1.060	0.0
PY-Dist	.999	0.016
LC-Point	1.042	0.0
LC-Dist	.979	0.021

C.2 Implied elasticities

Here are the results for the seven estimated points for the discretized uniform distribution and the 7 implied elasticies for each of the variations of the model matching 2013 SCF data.

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.920	5.263	0.872	3.911
0.947	6.387	0.901	4.849
0.973	8.003	0.944	6.228
0.999	10.524	0.979	8.460
1.027	15.006	1.015	12.685
1.053	25.192	1.051	23.720
1.080	71.032	1.086	127.148

C.3 Untargeted moments

These three tables present the wealth moments by age cohort for the 2013 wave of the SCF 12 , and the simulated version of these untargeted moments for the life cycle version of the model without heterogeneity 13 and then with heterogeneity 14.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.1391	-0.1376	-0.106	-0.0006
30-40	-0.0268	-0.0199	0.0097	0.1179
40-50	-0.0092	-0.0012	0.0318	0.1355
50-60	-0.0026	0.0078	0.0486	0.1621
60-70	-0.002	0.0141	0.0605	0.1714

Figure 12: Empirical Lorenz Curve Targets from the 2013 SCF.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0446	0.1342	0.2769	0.5248
30-40	0.0414	0.1412	0.2966	0.5306
40-50	0.0456	0.1398	0.285	0.509
50-60	0.05	0.144	0.2861	0.5051
60-70	0.0463	0.1345	0.2696	0.4826

Figure 13: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0162	0.0803	0.196	0.4152
30-40	0.0059	0.0383	0.1136	0.2946
40-50	0.0033	0.0204	0.0646	0.2117
50-60	0.0026	0.0141	0.046	0.1809
60-70	0.0022	0.0117	0.0416	0.1698

Figure 14: Simulated Untargeted Moments with Heterogeneity (R-dist).

D Results from 2016 wealth data

D.1 Estimated distribution of returns

Here are the results from the four versions of the model which makes the simulated wealth moments closest to the wealth moments measured in the 2016 survey.

	Mean	St. Dev
PY-Point	1.060	0.0
PY-Dist	.993	0.017
LC-Point	1.042	0.0
LC-Dist	.972	0.023

D.2 Implied elasticities

Here are the results for the seven estimated points for the discretized uniform distribution and the 7 implied elasticies for each of the variations of the model matching 2016 SCF data.

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.905	4.764	0.856	3.579
0.934	5.810	0.894	4.460
0.963	7.319	0.933	5.762
0.993	9.687	0.972	7.879
1.022	13.940	1.010	11.927
1.051	23.816	1.049	22.755
1.080	72.208	1.086	145.320

D.3 Untargeted moments

These three tables present the wealth moments by age cohort for the 2016 wave of the SCF 15 , and the simulated version of these untargeted moments for the life cycle version of the model without heterogeneity 16 and then with heterogeneity 17.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.2713	-0.2771	-0.2315	-0.0429
30-40	-0.0385	-0.0304	0.0111	0.1273
40-50	-0.0085	0.001	0.0441	0.1605
50-60	-0.0017	0.0071	0.0384	0.1209
60-70	0.0007	0.0144	0.0542	0.1574

Figure 15: Empirical Lorenz Curve Targets from the 2016 SCF.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0445	0.1342	0.2768	0.5247
30-40	0.0413	0.141	0.2963	0.5303
40-50	0.0455	0.1395	0.2845	0.5086
50-60	0.0499	0.1438	0.2859	0.5048
60-70	0.0462	0.1344	0.2694	0.4824

Figure 16: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0142	0.0755	0.1882	0.4052
30-40	0.0049	0.0347	0.1057	0.2801
40-50	0.0027	0.0182	0.0589	0.1965
50-60	0.0022	0.0123	0.0407	0.1647
60-70	0.0018	0.01	0.036	0.1534

Figure 17: Simulated Untargeted Moments with Heterogeneity (R-dist).

E Results from 2019 wealth data

E.1 Estimated distribution of returns

Here are the results from the four versions of the model which makes the simulated wealth moments closest to the wealth moments measured in the 2019 survey.

	Mean	St. Dev
PY-Point	1.060	0.0
PY-Dist	.999	0.016
LC-Point	1.042	0.0
LC-Dist	.978	0.021

E.2 Implied elasticities

Here are the results for the seven estimated points for the discretized uniform distribution and the 7 implied elasticies for each of the variations of the model matching 2019 SCF data.

PY		LC	
Estimated returns	Implied elasticities	Estimated returns	Implied elasticities
0.923	5.204	0.876	3.868
0.950	6.319	0.911	4.796
0.976	7.923	0.946	6.161
1.002	10.426	0.981	8.369
1.028	14.882	1.016	12.544
1.054	25.035	1.051	23.430
1.080	71.174	1.086	123.510

E.3 Untargeted moments

These three tables present the wealth moments by age cohort for the 2019 wave of the SCF 18 , and the simulated version of these untargeted moments for the life cycle version of the model without heterogeneity 19 and then with heterogeneity 20.

Empirical Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	-0.2136	-0.219	-0.1849	0.0075
30-40	-0.045	-0.0363	0.0093	0.1432
40-50	-0.0075	0.005	0.0478	0.1448
50-60	-0.0007	0.0112	0.0469	0.1358
60-70	-0.0004	0.012	0.0529	0.1562

Figure 18: Empirical Lorenz Curve Targets from the 2019 SCF.

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0445	0.1342	0.2767	0.5246
30-40	0.0412	0.1407	0.296	0.53
40-50	0.0453	0.1392	0.2841	0.5082
50-60	0.0498	0.1436	0.2856	0.5045
60-70	0.0462	0.1343	0.2692	0.4822

Figure 19: Simulated Untargeted Moments without Heterogeneity (R-point).

Simulated Lorenz Shares (10-Year)

age	20th	40th	60th	80th
25-30	0.0159	0.0799	0.1953	0.4143
30-40	0.0058	0.038	0.1128	0.293
40-50	0.0033	0.0202	0.0641	0.2098
50-60	0.0025	0.0139	0.0454	0.1788
60-70	0.0021	0.0115	0.0409	0.1679

Figure 20: Simulated Untargeted Moments with Heterogeneity (R-dist).