Chapter 3

The Case for a Measure of Wealth Inequality Aversion

Abstract

The inequality literature is notable in its ability to bring together four distinct concepts: (i) measures of inequality, (ii) social welfare functions, (iii) mean values, and (iv) models of choice under uncertainty, as noted by Dalton 1920. This interplay between statistical and theoretical analysis has been commonly applied in the context of inequality in the distribution of *income*. It is well-known that income and wealth inequality are markedly distinct phenomena, and this distinction becomes more pronounced when racial considerations are central. The literature has long documented that wealth is more strongly skewed than income, especially in the upper tail (Benhabib and Bisin 2018). Notably, as recent as 2019, it has been reported that "the median white household held $188,200 in wealth – 7.8 times that of the typical Black household". This paper provides a theoretical foundation for wealth inequality that incorporates the first three of the four distinct concepts. The ranking over wealth distributions is established through a choice under ambiguity model, and the *social welfare approach* to inequality measures in Atkinson 1971 is applied in this setting to derive

a measure of wealth inequality aversion. Using the triennial data on wealth in the Survey of Consumer Finances (SCF) from 1989–2022, I show that the choice under objective uncertainty inequality measure underestimates wealth inequality compared to the measure proposed via choice under ambiguity. Both theoretical measures of inequality aversion track the general trend in the Black-White median wealth gap over the same time period.

3.1 Introduction

The distribution of outcomes such as income and wealth among individuals in a society are of interest not only to economists, but to those interested in social welfare and inequality. As Hong 1983 notes, the literature on inequality notably connects four distinct concepts; (i) measures of inequality, (ii) social welfare functions, (iii) mean values, and (iv) models of choice under uncertainty.

Measures of inequality, such as the variance and the coefficient of variation, represent the traditionally statistical approach to characterizing inequality in a society. Almost a century ago, Dalton 1920 suggested that a given measure of inequality would correspond to a social welfare function. This can be gleaned from the fact that different measures of inequality produce different rankings over distributions.

The contribution of Atkinson 1970 is two-fold. First, he provides a theoretical foundation for ranking distributions and a notion of income inequality aversion by connecting results from the choice under risk literature with Dalton's observation of underlying social welfare functions. Although not explicitly stated, he later proposes an inequality measure which highlights the connection between measures and quasilinear mean values. This is done through the use of an analogy to certainty equivalence from the choice under risk literature.

This literature is almost exclusively interested in characterizing inequality in the distribution of income. In the papers that claim to refer to wealth inequality, there is no "true" distinction, since the underlying choice under uncertainty model used only incorporates objective uncertainty. Since the literature is solid on the differences between skewness in the distribution of income and the distribution of wealth, one may wish to produce an inequality measure which makes a similar distinction. This measure can provide an alternative lens to view the evolution of racial disparities over time, much like summary statistics such as the racial wealth gap.

The goal of this paper is to produce an alternative measure inequality in the distribution of wealth using this *social welfare approach* to be compared to conventional summary statistics of racial wealth inequality, such as the black-white median wealth gap. This is achieved by exploiting connections between the aforementioned distinct concepts. I make use of an analogy between uncertainty aversion and wealth inequality aversion through specifications on the underlying social welfare function in a choice under ambiguity model, as well as the familiar *equally distributed measure* of an outcome to reach a measure of wealth inequality aversion, in the sense that it takes into consideration particular features of wealth accumulation and distribution which make it different from other distributional outcomes (like income).

The paper proceeds as follows. Section 3.2 presents arguments for distinguishing income and wealth inequality in this framework. Sections 3.3–3.5 then apply the approach to wealth, in the following order: choice under objective–subjective uncertainty, the social welfare approach, and the resulting inequality measure. Subsection 3.6 provides the SCF-based computational implementation and comparison to the Black-White median wealth gap. Section 3.7 concludes. The appendix A collects proofs and provides an explicit formulation of the construction towards the measure of income inequality aversion from the literature.

3.2 Motivation

The goal of this section is to argue that *there are fundamental features about wealth that make it different from income when it comes to ranking distributions.* The arguments I provide are a blend of empirical facts as well as a thought experiment and logical argument. Then the core argument of this paper becomes: **if i) there is a serious difference between ranking income distributions and ranking wealth distributions, and ii) economic theory can be used to produce a measure of income inequality using the choice under uncertainty literature, then the same approach can be used to produce a measure of wealth inequality using the choice under ambiguity literature**.

3.2.1 Empirical facts about income and wealth

It is well-known that income data is more reliably collected than wealth data. Consider a quote from Saez and Zucman 2014 which provides an estimate for wealth inequality using available administrative data:

> Because of the lack of administrative data on wealth, none of the existing sources offer a definitive estimate. We see our paper as an attempt at using the most comprehensive administrative data currently available, but one that ought to be improved in at least two ways: (i) by using additional information already available at the Statistics of Income division of the IRS, and (ii) new data that the US Treasury could collect at low cost. A modest data collection effort would make it possible to obtain a better picture of the joint distributions of wealth, income, and saving, a necessary piece of information to evaluate proposals for consumption or wealth taxation.

So it is clear that here are differences in the collection of income data and wealth data. But is it enough to justify incorporating modeling choices from the choice under subjective uncertainty framework in the exercise of comparing wealth distributions?[1]

Consider next a quote from Bricker et al. 2016 which provides a justification for their use of administrative data and their measurement of income and wealth inequality:

[1]in combination with the choice under objective uncertainty is what is referred to as "choice under ambiguity" framework.

> In general, administrative data should provide better estimates of top income and wealth shares, because traditional random household surveys suffer from underrepresentation of wealthy families. Unlike most other household surveys, the SCF is designed to overcome the underrepresentation problem, because administrative data are used to select the sample, and rigorous targeting and accounting for wealthy family participation assures those families are properly represented in the survey data.

Not only is "good" data on income more accessible, but the estimation of wealth shares typically employs some combination of administrative and survey data. But survey data is by, quite literally, a subjective measure: it implicitly relies on individual's truthful and accurate reporting of their own levels of wealth.

3.2.2 A thought experiment

Country A has 60 individuals; there are two possible races so that each individual is either black or white. Suppose there are 10 black and 50 white individuals. There is an initial wealth distribution $f(w)$, so that $w = (w_1, \ldots, w_{60})$ and $f(w_i)$ gives the proportion of individuals with that level of wealth. One can consider total wealth, $\bar{w} = \sum_{i=1}^{60} w_i f(w_i)$, as well as group wealth totals, $\bar{w}_b$ and $\bar{w}_w$.

Partition each racial group into rich and poor individuals:

$$B = B^R \cup B^P, \quad W = W^R \cup W^P,$$

with $|B^R| = 5,\ |B^P| = 5,\ |W^R| = 25,\ |W^P| = 25$.

A policymaker compares two budget-balanced redistributive policies:

> **Policy A:** decrease wealth of each individual in B^R by 20 units, and increase wealth of each individual in W^P by 4 units.

Policy B: decrease wealth of each individual in W^R by 4 units, and increase wealth of each individual in B^P by 20 units.

Both policies transfer the same total amount of wealth:

$$5 \times 20 = 25 \times 4 = 100.$$

Assuming that, aside from racial labels, the post-policy wealth profile is the same under both policies, the choice under objective uncertainty framework would treat Policies A and B as equivalent, since they induce the same resulting wealth distribution. In other words, they have the *same effect on wealth inequality under this framework*.

However, it is not unreasonable to suspect that these two policies would not receive the same support in practice, especially in the U.S. context where race is tied to historically persistent barriers to wealth accumulation. This is the sense in which the choice under objective uncertainty framework, while useful for income distributions, may miss relevant features when ranking wealth distributions. If this is the case, then a notion of subjective uncertainty can be incorporated in a straightforward way: denote the finite state space $S = \{\text{black}, \text{white}\}$. From here, using a model that allows for uncertainty aversion is a reasonable deviation from the objective case.

3.2.3 A logical argument

An immediate counterargument to the previous thought experiment is that the factors which make it more difficult for blacks to accumulate wealth in the U.S. (which is the assumed reasoning behind not being indifferent between the two policies) should also apply to blacks earning income. The first rebuttal to this point is the aforementioned observable data on differences between income and wealth inequality.

However, I believe an appeal to the underlying meaning of the word "wealth" statements of the late philosopher John Rawls may convince readers that income inequality and wealth inequality truly are different phenomena, and that the latter has a objective-subjective uncertainty structure that makes it suited for the choice under ambiguity model to produce an accompanying measure of wealth inequality aversion.

Rawls 1971 is interested in the problem of getting a group of people with different circumstances and motives to agree on a *social contract*; that is, an agreement on a system of governing for which all members of the group must abide by.

He proposes that this problem should be approached as if those deciding on the governance of society are behind a *veil of ignorance*: decision-makers are ignorant of their own circumstances. He then proposes two principles that may accompany this veil of ignorance in the delivery of justice for institutions which govern our society. Consider Rawls' final statement of the two principles of justice for institutions:

1. Each person is to have an equal right to the most extensive total system of equal basic liberties compatible with a similar system of liberty for all.

2. Social and economic inequalities are to be arranged so that they are both:

   (a) to the greatest benefit of the least advantaged, consistent with the just savings principle, and

   (b) attached to offices and positions open to all under conditions of fair equality of opportunity.

But how is this related to the social planner's decision problem of ranking wealth distributions? The link is that there is some **underlying notion of a social contract implicit in any concept of wealth, since wealth requires enforceable ownership rights.** In modern economies, those rights are defined and protected by a government (or governing legal authority): individuals must agree on what it means to own something, and must respect that some individuals own more and/or less than they do. By contrast, income is often generated through a worker–employer contract, which the employer may legally modify or terminate (subject to labor law and contract terms). This makes wealth inequality more directly tied to legal and political institutions than labor income[2].

Consider another quote from Rawls, which is almost exactly the intuitive counterpart to the modeling choices I make later in the characterization of a comparison over wealth distributions:

> The maximin rule tells us to rank alternatives by their worst possible outcomes: we are to adopt the alternative the worst outcome of which is superior to the worst outcomes of the others[3].

3.3 Ranking wealth distributions using choice under ambiguity

Consider the excerpt from Gilboa 2009 about the choice under ambiguity model, which motivates the modeling choices that will be made to characterize a ranking over wealth distributions[4]:

> To make sure that we understand the structure, observe that there are two sources of uncertainty: the choice of the state s, which is sometimes referred to as "subjective uncertainty", because no objective probabilities are given on it, and the choice of y, which is done with objective probabilities once you chose your act and Nature chose a state. Specifically, if you choose $p \in L$ and Nature chooses $s \in S$, a roulette wheel is spun, with distribution p_s over the outcomes Y, so that your probability to get outcome y is $p_s(y)$.

[2]Further arguments could be made that income suffers from legal and political contamination as well.

[3]Coincidentally, Rawls' second principle of justice for institutions is often considered by economists to be interchangeable with the maximin principle, despite his belief that it is "undesirable to use the same name for two things that are so distinct.".

[4]I move away from the notation used there in favor of the "choice over lotteries" notation to tie into the birth lottery analogy made earlier in the paper

3.3.1 Analytical framework

Denote the set of outcomes $Y = [0, \bar{y}]$. Then, the choice set is given by:

$$L = \left\{ p : 2^{[0,\bar{y}]} \to \mathbb{R} \mid p(\cdot) \text{ is an income frequency distribution} \right\}.$$

We may define a binary relation over both sets Y, L :

$$\succsim_y \subseteq [0, \bar{y}] \times [0, \bar{y}] \subseteq \mathbb{R} \times \mathbb{R}$$

$$\succsim \subseteq L \times L.$$

Notice that $\succsim_y$ may be represented by a real-valued utility function. This is the social welfare $U(y)$ which is a key object of analysis in this paper.

Preliminary remarks

We are concerned with the comparison of two frequency distributions $f(w)$ of an outcome w which we refer to as wealth. We seek to use the notion of *uncertainty aversion* as an analogy to the use of the notion of risk aversion in the characterization of a ranking over income distributions.

The presence of both objective and subjective uncertainty is at the heart of this analysis. Section 2 covered the analysis for objective uncertainty. Thus, the wealth frequency distribution $f(w)$ must be formalized in this abstract setting so that it explicitly captures both forms of uncertainty. Namely, each wealth distribution is associated with some relevant, underlying state space S (the source of subjective uncertainty), and its objective component $p(y) \in L$.

This suggests that the objects of interest (rankable wealth distributions $f(w)$) are a family of distributions $p(y) \in L$, indexed by elements of the state space $s \in S$, or:

$$\{p_s(y)\}_{s\in S}.$$

Thus, we may view each act $f \in F$ as a "state-contingent frequency distribution". Moreover, upon fixing a state $s' \in S$, the analysis will become identical to the use of the choice under objective uncertainty model that was used to reach a ranking over income frequency distributions.

With this in mind, we work under the following assumption on the functional form of wealth distributions for the remainder of this paper:

Assumption 1. *Each wealth distribution $f(w)$ can be written as $p_s(y)$.*

3.3.2 Axiomatization

AA 1 (Weak Order). *$\succsim$ is complete and transitive.*

AA 2 (Continuity). *For every $f, g, h \in F$, if $f \succ g \succ h$, there exists $\alpha, \beta \in (0,1)$ such that*

$$\alpha f + (1-\alpha)h \succ g \succ \beta f + (1-\beta)h.$$

C-Independence (C-Independence). *For every $f, g \in F$, every constant $h \in F$ and every $\alpha \in (0,1)$,*

$$f \succsim g \iff \alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h.$$

AA 3 (Monotonicity). *For every $f, g \in F$, $f(s) \geq g(s)$ for all $s \in S$ implies $f \geq g$.*

AA 4 (Non-trivality). *There exists $f, g \in F$ such that $f \succ g$.*

Uncertainty Aversion (Uncertainty Aversion). *For every $f, g \in F$, if $f \sim g$, then, for every $\alpha \in (0,1)$,*

$$\alpha f + (1-\alpha)g \succsim f.$$

3.3.3 Expected utility representation of the ranking over wealth distributions

Finally, we have the representation theorem by Gilboa and Schmeidler 1989.

Theorem 1. *$\succsim$ satisfies AA1, AA2, C-Independence, AA4, AA5, and Uncertainty Aversion if and only if there exists a closed and convex set of probabilities on S, $C \subseteq \Delta(S)$, and a non-constant function $U : Y \to \mathbb{R}$ such that, for every $f, f^* \in F$,*

$$f \succsim f^* \iff \min_{\lambda \in C} \int_S \big(\mathbb{E}_{p_s}[U]\big)\, d\lambda \geq \min_{\lambda \in C} \int_S \big(\mathbb{E}_{p_s^*}[U]\big)\, d\lambda.$$

From here on out, we assume that wealth distributions will be ranked according to:

$$W^A \equiv \min_{\lambda \in C} \int_S (\mathbb{E}_{p_s}[U]) d\lambda$$

$$= \min_{\lambda \in C} \int_S \int_0^{\bar{y}} p_s(y) U(y) dy d\lambda.$$

Notation. Superscript A denotes the ambiguity specification; this is the first appearance of W^A in the main text.

3.4 The social welfare approach to ranking wealth distributions

3.4.1 Partial ranking over wealth distributions

From here, I assume that the function $U : Y \mapsto \mathbb{R}$ is an increasing and concave function. Again, we seek the conditions for which a ranking over wealth distributions can be achieved without any further specifications on $U(y)$.

Recall that the second-order stochastic dominance result was necessary and sufficient to reach a partial ordering over income distributions. The proposition below provides a sufficient condition that extends this logic to the ambiguity setting with maxmin aggregation over priors.

Proposition 1. *Define*

$$W^A(\{p_s\}; U) \equiv \min_{\lambda \in C} \int_S \left(\mathbb{E}_{p_s}[U] \right) d\lambda, \quad C \subseteq \Delta(S).$$

Notation. The notation W^A was introduced in Section 3.3. If, for every $s \in S$,

$$\int_0^z [P_s(y) - P_s^*(y)] \, dy \leq 0 \quad \text{for all } z, \; 0 \leq z \leq \bar{y},$$

where

$$P_s(y) = \int_0^y p_s(t) \, dt \quad \text{and} \quad P_s^*(y) = \int_0^y p_s^*(t) \, dt,$$

then p is weakly preferred to p^* according to W^A for all $U(y)$ with $U' > 0$ and $U'' \leq 0$.

The proof can be found in the Appendix. In general, the converse need not hold under a maxmin criterion unless additional assumptions are imposed on C.

3.4.2 Complete ranking over wealth distributions

To reach our complete ordering over wealth distribution, we must guarantee that $U(y)$ is specified up to a linear (monotonic) transformation. Work done by Kihlstrom and Mirman 1981 extending the notion of relative risk aversion results to the case of "multidimensional commodities" implies that the restriction we are after is the class of social welfare functions representing homothetic preferences. That is,

$$U(y) = \begin{cases} \alpha + \beta \frac{y^{1-\epsilon}}{1-\epsilon}, & \epsilon \neq 1 \\ \alpha + \beta \ln(y), & \epsilon = 1 \end{cases} \tag{3.1}$$

For strict concavity in this class, take $\beta > 0$, $y > 0$, and $\epsilon > 0$ (with $\epsilon = 1$ giving the log case); note that $\epsilon = 0$ yields linear utility.

It is well-known that, in the risk and risk aversion literature, this functional form is associated with the class of utility functions representing constant relative risk averse (CRRA) and decreasing absolute risk averse (DARA) preferences.

3.5 Proposing a measure of wealth inequality aversion

The ranking over wealth distributions via specifications on $U(y)$ will allow us to propose measures of inequality. To see the novelty in this "social welfare approach", notice that measures of inequality are generally statistical objects. Namely, the variance, coefficient of variation, and mean deviation are each calculated using collected data on the distribution of an outcome such as income or wealth.

Consider one plausible inequality measure: given some present distribution $f(w) = \{p_s(y)\}_{s\in S}$, the ratio between social welfare at the current distribution and social welfare at equal wealth μ. Using the robust welfare index

$$W^A \equiv \min_{\lambda\in C} \int_S \int_0^{\bar{y}} U(y)\, p_s(y)\, dy\, d\lambda, \qquad C \subseteq \Delta(S),$$

we may write

$$D^A = \frac{W^A}{U(\mu)}.$$

Notation. Superscript A denotes ambiguity objects; this is the first appearance of D^A in the main text.

However, this ratio is not invariant to positive affine transformations of U, so different cardinal representations can produce different values of D^A even when they induce the same ranking.[5]

[5] As shown by Dalton 1920 and others.

3.5.1 The equally distributed equivalent measure of inequality

To avoid this undesirable property, define the equally distributed equivalent wealth level as follows.

Definition 1. *The equally distributed equivalent level of wealth w_{EDE} is defined by*

$$U(w_{EDE}) = W^A.$$

Equivalently,

$$U(w_{EDE}) = \min_{\lambda \in C} \int_S \int_0^{\bar{y}} U(y)\, p_s(y)\, dy\, d\lambda.$$

Since each p_s integrates to one on Y and each $\lambda \in C$ is a probability on S, the normalization factor is one, so no extra scale term is needed in the definition.

The corresponding equally distributed equivalent inequality measure is

$$I^A = 1 - \frac{w_{EDE}}{\mu}.$$

Notation. This is the first appearance of I^A in the main text.

This measure is cardinally robust (with respect to positive affine transformations of U) and lies in $[0, 1]$ under the usual ordering conditions.

3.5.2 Closed-form expression under homothetic preferences

For computation, the key identity is

$$w_{EDE} = U^{-1}(W^A).$$

For the homothetic class,

$$U(y) = \begin{cases} \alpha + \beta \dfrac{y^{1-\epsilon}}{1-\epsilon}, & \epsilon \neq 1, \\ \alpha + \beta \ln y, & \epsilon = 1, \end{cases} \qquad \beta > 0,$$

with $y > 0$ and $\epsilon > 0$ for strict concavity (note: $\epsilon = 0$ is linear and therefore not strictly concave).

If $\epsilon \neq 1$,

$$w_{EDE} = \left[\frac{1-\epsilon}{\beta}\left(W^A - \alpha\right)\right]^{\frac{1}{1-\epsilon}}, \qquad I^A = 1 - \frac{1}{\mu}\left[\frac{1-\epsilon}{\beta}\left(W^A - \alpha\right)\right]^{\frac{1}{1-\epsilon}}.$$

If $\epsilon = 1$,

$$w_{EDE} = \exp\left(\frac{W^A - \alpha}{\beta}\right), \qquad I^A = 1 - \frac{1}{\mu}\exp\left(\frac{W^A - \alpha}{\beta}\right).$$

Appendix A.3 gives the same derivation for the objective-uncertainty benchmark and then shows that the ambiguity case is obtained by replacing W^O with W^A.

3.6 Comparing inequality aversion measures to the racial wealth gap

3.6.1 Data

I use the triennial Survey of Consumer Finances (SCF), 1989–2022, and restrict the sample to Black and White households in each wave. For each year, I compute weighted medians for net wealth and gross assets, and then compute I^O and I^A under two treatments: (i) a restricted sample that keeps only strictly positive wealth values, and (ii) the full sample which adds a year-specific constant so that all observations shifted to be strictly positive.[6]

The *ambiguity* aggregator used for I^A follows the Gilboa–Schmeidler construction with

[6] Appendix Figure A.2 plots the shift constant and the weighted share with non-positive wealth.

$C = \Delta(S)$ and $S = \{\text{Black}, \text{White}\}$. Because the criterion is linear in probabilities over the two states, the inner minimum is attained by putting all weight on whichever racial group has the lower within-group expected utility that year, so computationally $W_t^A = \min\{\mathbb{E}_{B,t}[U(y)], \mathbb{E}_{W,t}[U(y)]\}$. The wedge $I_t^A > I_t^O$ arises relative to the objective benchmark W^O defined in Appendix A.3.

With these modeling objects fixed, the theoretical welfare expressions map directly into computable objects in the data: for each year and wealth concept, we compute W^O and W^A, invert U to recover the equally distributed equivalent wealth level, and then construct I^O and I^A. The remaining step is to set values for ϵ, which governs the curvature of $U(y)$ and therefore the degree of inequality aversion imposed in the computation.

The parameter ϵ indexes the curvature of the social welfare function $U(y)$ and therefore the strength of inequality aversion: larger ϵ means stronger concavity, a higher penalty on low-wealth outcomes, and thus higher implied inequality aversion in both I^O and I^A.

For orientation, read the CRRA index in four regions: (i) $\epsilon = 0$ is the affine (linear) benchmark with no inequality aversion in the objective aggregator, reported separately in Appendix A.6; (ii) $0 < \epsilon < 1$ yields concavity that is still milder than the logarithmic case; (iii) $\epsilon = 1$ is the log boundary; (iv) $\epsilon > 1$ imposes tail sensitivity strictly stronger than the log case. The subsections below walk through $\epsilon \in \{0.25, 0.5, 1, 1.5, 2\}$ so that each reported value sits in a clearly labeled part of this map.

3.6.2 The Black-White gap in net wealth and gross assets

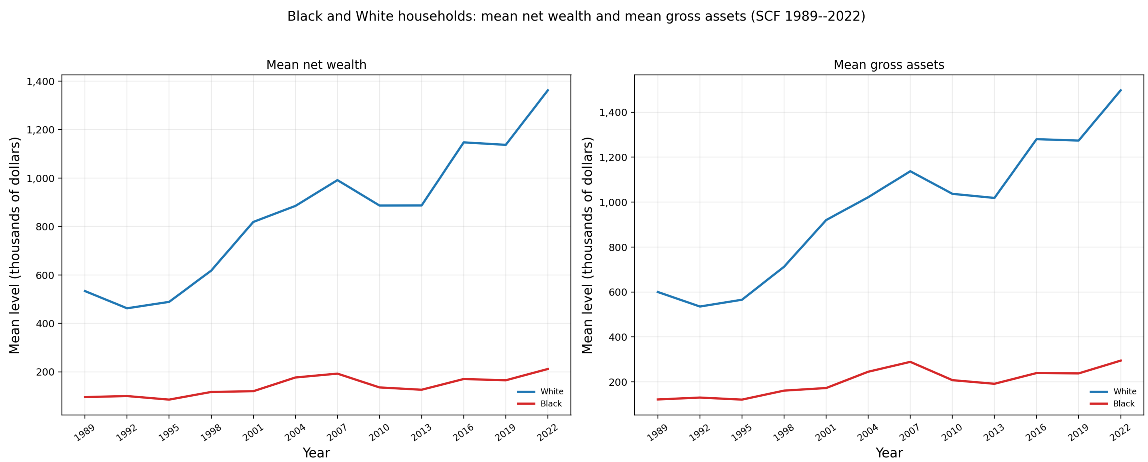

Figure 3.1: Mean net wealth (left panel) and mean gross assets (right panel) for Black and White households, SCF 1989–2022.

Figure 3.1 plots race-specific means before differencing: White household means lie above Black means in both net wealth and gross assets throughout the sample.

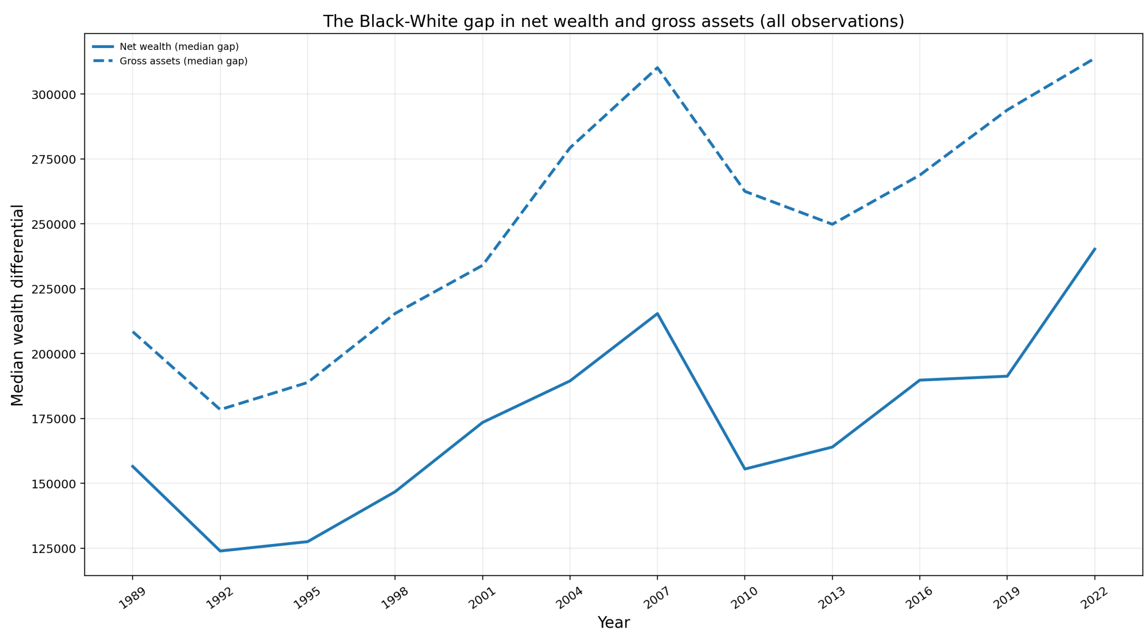

Figure 3.2: The Black-White gap in net wealth and gross assets: median wealth differential, SCF 1989–2022.

Figure 3.2 shows that the median Black-White differential is positive and persistent in both series, with gross-asset gaps generally larger than net-wealth gaps over the sample.

Checking gross assets alongside net wealth is useful because it verifies that the empirical trend is not an artifact of liability measurement alone: both series move with similar broad dynamics over time. At the same time, the gross-asset gap is higher in levels than the net-wealth gap, which is consistent with definition since net wealth subtracts debts while gross assets do not.

3.6.3 Low concavity: $\epsilon = 0.25$

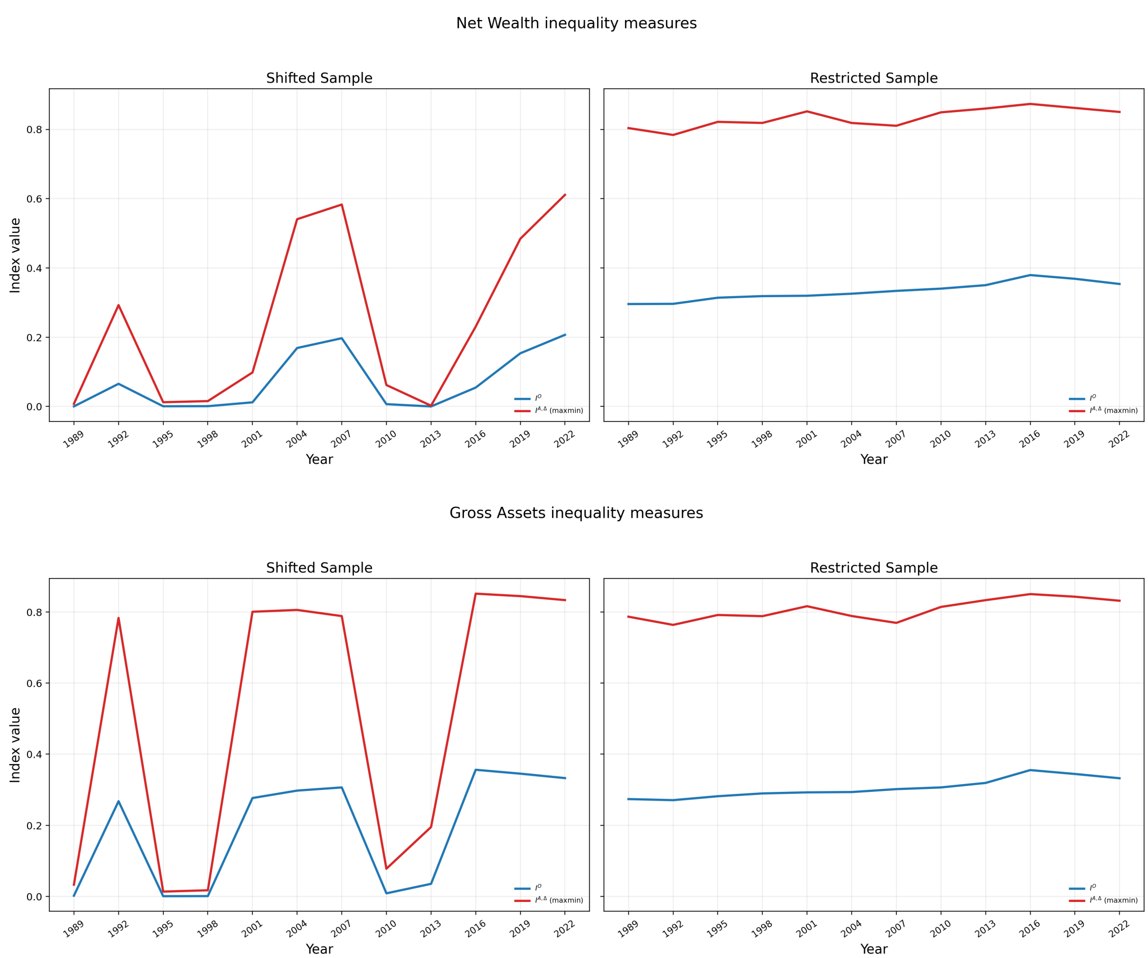

Figure 3.3: Inequality version measures for $\epsilon = 0.25$ (top: net wealth, bottom: gross assets, left: shifted sample, right restricted sample).

Even with a low level of inequality aversion (curvature near zero), the ambiguity ranking already yields higher inequality levels than the objective ranking in both wealth concepts

$(I^A > I^O)$. When I restrict the sample to respondents with positive wealth only, inequality is extremely high and stable across time for the wealth inequality aversion measure. For the full sample (shifted so that everyone has positive wealth), there is more variation over the time period, with some years showing significantly low levels of inequality. The key takeaway is that *the inequality aversion measure reached using the choice under objective uncertainty framework consistently reports that inequality is lower than it is under the ambiguity aversion measure.*

3.6.4 Mild concavity: $\epsilon = 0.5$

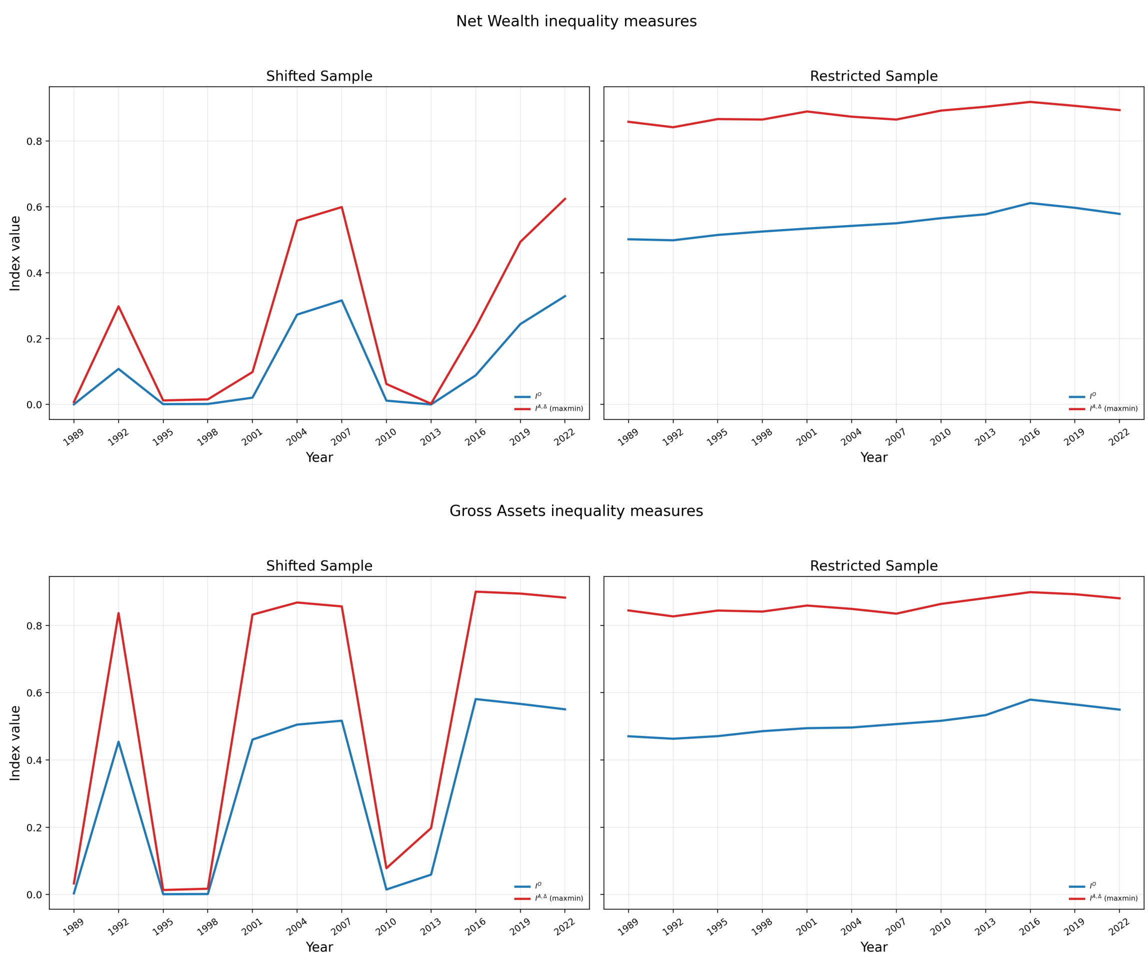

Figure 3.4: Inequality aversion measures for $\epsilon = 0.5$.

For $\epsilon = 0.5$, the performance of the wealth inequality aversion measure I^A in the restricted sample is the same: high and comparatively stable inequality levels over time. This time, the inequality aversion measure I^O is now higher for this higher value of epsilon. Again, the shifted sample shows more variation across year, while retaining the feature that I^A is consistently above I^O; so the finding that the objective measure understates inequality relative to the ambiguity measure is preserved.

3.6.5 Logarithmic case: $\epsilon = 1$

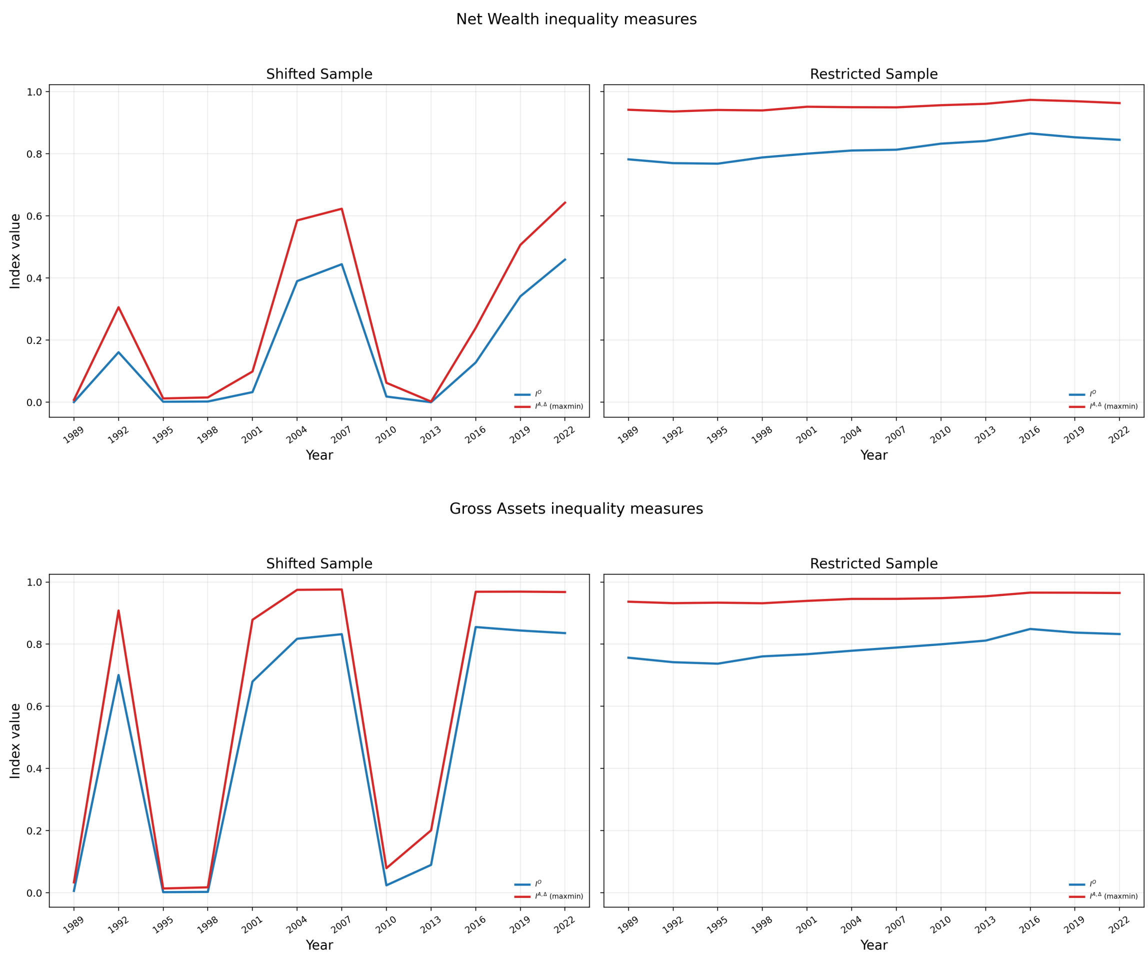

Figure 3.5: Inequality aversion measures for $\epsilon = 1$.

The same pattern holds $\epsilon = 1$. For both samples, $I^A > I^O$, and in the restricted-sample, the objective measure I^O has moved even closer to I^A. An interesting observation throughout these figures is that, for the shifted sample, both measures consistently report higher inequality levels in gross assets than in net wealth, especially between the years 1998-2010. This pattern is robust to values of ϵ.

3.6.6 Strong concavity: $\epsilon = 1.5$

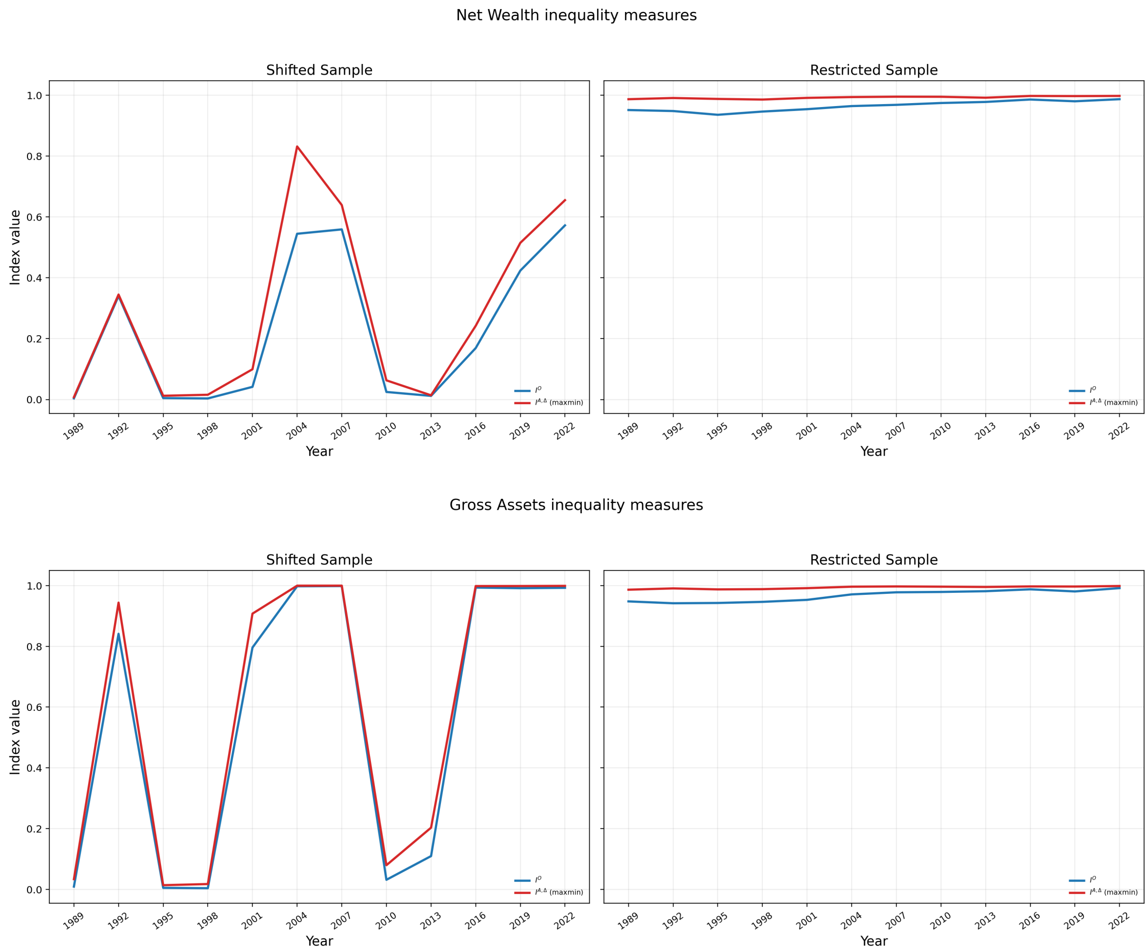

Figure 3.6: Inequality aversion measures for $\epsilon = 1.5$.

At this level of concavity in $U(y)$, the restricted sample shows that the measures I^A and I^O track each other very closely. Both are extremely high and stable over time. The pattern persists for the shifted sample, but reported levels of inequality are significantly larger here (for gross assets, there are multiple periods where inequality is essentially 1.). Still, the finding that the objective uncertainty aversion measure I^O underestimates inequality compared to the ambiguity aversion measure I^A is preserved.

3.6.7 Strongest curvature: $\epsilon = 2$

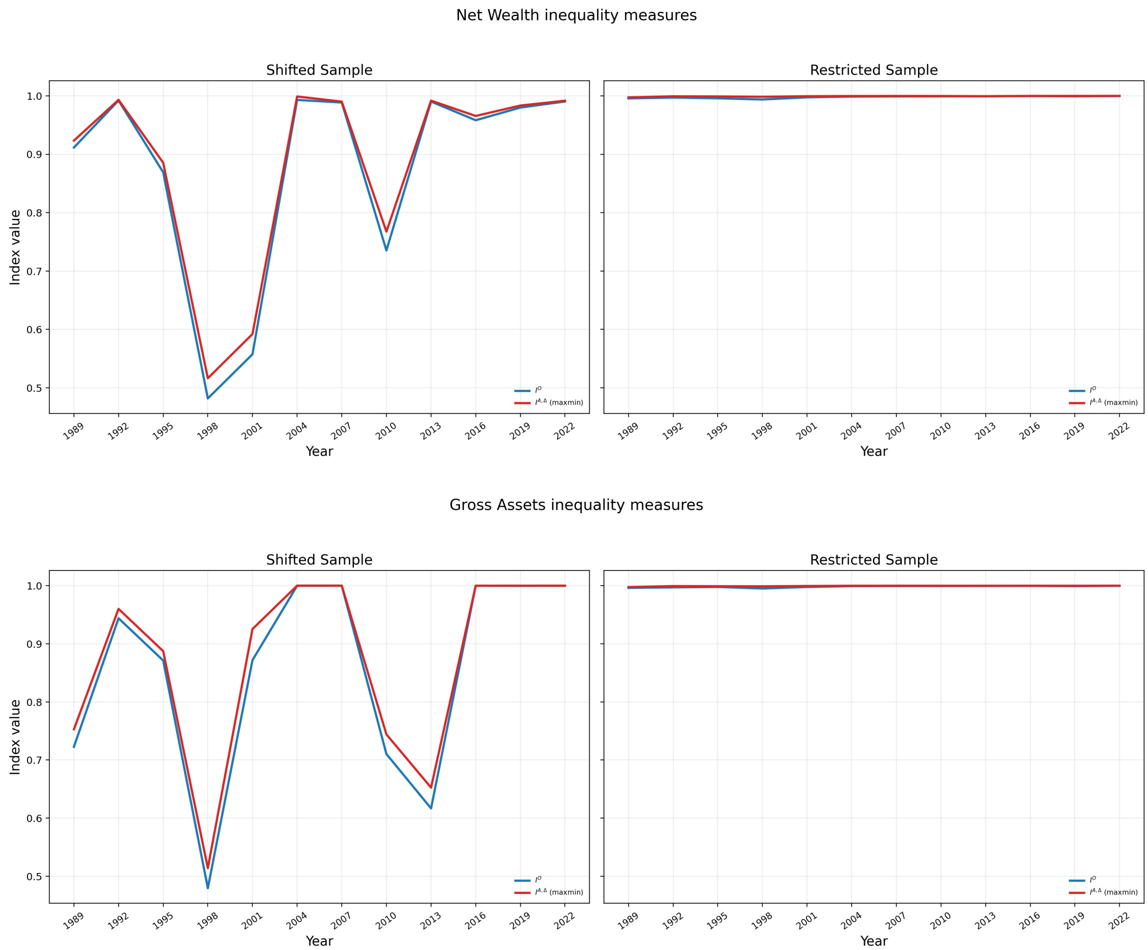

Figure 3.7: Inequality aversion measures for $\epsilon = 2$.

This is the largest value of ϵ reported in the main text; it is the most averse to inequality in the sense that the social welfare functional penalizes low wealth sharply.

As ϵ rises from 1.5 to 2, the restricted sample essentially shows no difference between I^A and I^O. The variation in inequality over time in the shifted sample is the most pronounced here, and are almost identical for net wealth and gross assets.

Final takeaways

Taken together, the inequality-measure series tell the same within-sample story across specifications (from $\epsilon = 0.25$ through $\epsilon = 2$): rising (or at the very least, extremely high) inequality over the time period. Interestingly, when we restrict the sample to positive wealth values, the inequality aversion measures I^A and I^O are stable over time. However, Figure A.1 shows that net-wealth and gross-assets median gaps still vary over time and trend upward. This suggests that the theoretical indices are unusually stable when applied to the restricted sample. Additionally, in the restricted sample, the wealth inequality aversion measure I^A is especially robust; across reported ϵ values, it is stable over time and very close to 1.

By contrast, in the shifted sample, treatment both I^O and I^A are more variable over time and show episodes of comparatively low measured inequality at low and moderate ϵ; they begin to pull upward noticeably at $\epsilon = 1.5$, and become frequently near one only in the highest-curvature case $\epsilon = 2$. This higher time variation in the shifted series tracks the median-gap dynamics in Figure 3.2 much more closely than the restricted-sample series.

3.7 Conclusion

We have reached an alternative measure of wealth inequality, I^A. The measure both utilizes the notion of representative wealth levels and preserves the mean-independence property that many statistical measures of inequality have. The goal of this exercise is to apply it in the context of inequality in racial wealth. There, the story is generally told using the aforementioned statistical measures of inequality (most notable, the *black-white median wealth gap*).

With the arguments developed in Sections 3.2 and 3.3, it seems reasonable to suspect that a state space such as $S = \{\text{black, white}\}$ should largely inform our measure of wealth inequality via some ranking over wealth distributions. Historical episodes such as enslavement and the era of Jim Crow laws suggest that race may be a socially relevant feature in characterizing the level of inequality in a given wealth distribution. The main issue is whether the effects of these historical episodes persist over time, as well as how explanatory they are for racial differences in wealth.

The SCF-based computational exercise in Section 3.6 delivers a clear result: the objective benchmark understates wealth inequality relative to the ambiguity-based measure. This comparison is robust for both net wealth and gross assets, and the model-based series follow the same broad time pattern as the Black-White median wealth-gap diagnostics; the restricted sample is higher and smoother, while the shifted sample is more time-varying.

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Appendix A

Supplementary Appendix

A.1 Proofs

A.1.1 Proposition 1

Proof. Let

$$W^A(\{p_s\}; U) \equiv \min_{\lambda \in C} \int_S \big(\mathbb{E}_{p_s}[U]\big)\, d\lambda, \qquad C \subseteq \Delta(S),\ C \neq \emptyset,$$

and fix any U with $U' > 0$ and $U'' \leq 0$.

Suppose that, for every state $s \in S$,

$$\int_0^z \big[P_s(y) - P_s^*(y)\big] dy \leq 0 \quad \text{for all } z \in [0, \bar{y}],$$

where

$$P_s(y) = \int_0^y p_s(t)\, dt, \qquad P_s^*(y) = \int_0^y p_s^*(t)\, dt.$$

For each fixed state s, this is exactly the single-state second-order stochastic dominance condition. Hence,

$$\mathbb{E}_{p_s}[U] \geq \mathbb{E}_{p_s^*}[U] \quad \text{for every } s \in S.$$

Let $\lambda \in C$ be arbitrary. Since λ is a probability measure on S,

$$\int_S \mathbb{E}_{p_s}[U]\, d\lambda \geq \int_S \mathbb{E}_{p_s^*}[U]\, d\lambda.$$

Because this inequality holds for every $\lambda \in C$, taking minima over C gives

$$\min_{\lambda \in C} \int_S \mathbb{E}_{p_s}[U]\, d\lambda \geq \min_{\lambda \in C} \int_S \mathbb{E}_{p_s^*}[U]\, d\lambda.$$

Therefore,

$$W^A(\{p_s\}; U) \geq W^A(\{p_s^*\}; U).$$

Since U was arbitrary in the class $\{U : U' > 0,\ U'' \leq 0\}$, the claim follows.

$\square$

Remark 1. *The argument above establishes a sufficient condition. In general, the converse fails under a maxmin aggregator: the minimizing prior may assign zero weight to states where state-by-state dominance fails. An equivalence requires stronger assumptions (for example, comparisons for every λ in a sufficiently rich set of priors that identifies each state).*

A.2 Single-state benchmark

Proposition 2. *For a single-state problem with distributions f and f^* on $[0, \bar{y}]$, define*

$$F(y) = \int_0^y f(t)\, dt, \qquad F^*(y) = \int_0^y f^*(t)\, dt.$$

Then

$$\mathbb{E}_f[U] \geq \mathbb{E}_{f^*}[U] \quad \text{for all } U \text{ with } U' > 0,\ U'' \leq 0$$

if and only if

$$\int_0^z \big[F(y) - F^*(y)\big]\, dy \leq 0 \quad \text{for all } z \in [0, \bar{y}].$$

Moreover, strict preference for all such U requires strict inequality for some $z \in (0, \bar{y})$.

A.3 The Analogy between Income and Wealth Inequality Aversion

To understand the problem, I begin with describing the model which proposes measures of income inequality using the choice under objective uncertainty literature.

Analytical framework

Denote the set of outcomes $Y = [0, \bar{y}]$. Then, the choice set is given by:

$$L = \left\{ p : 2^{[0,\bar{y}]} \to \mathbb{R} \mid p(\cdot) \text{ is an income frequency distribution} \right\}.$$

We may define a binary relation over both sets Y, L :

$$\succsim_y \subseteq [0, \bar{y}] \times [0, \bar{y}] \subseteq \mathbb{R} \times \mathbb{R}$$

$$\succsim \subseteq L \times L.$$

Notice that $\succsim_y$ may be represented by a real-valued utility function. This is the social welfare $U(y)$ which is a key object of analysis in this paper.

Axiomatization

V 1 (Weak Order)**.** $\succsim$ *is complete and transitive.*

V 2 (Continuity)**.** *For every $p(\cdot)$, $p^*(\cdot)$, $p'(\cdot) \in L$, if $p(\cdot) \succ p^*(\cdot) \succ p'(\cdot)$, there exists $\alpha, \beta \in (0, 1)$ such that*

$$\alpha p + (1 - \alpha) p' \succ p^* \succ \beta p + (1 - \beta) p'.$$

V 3 (Independence)**.** *For every $p, p^*, p' \in L$ and every $\alpha \in (0,1)$ such that*

$$p \succsim p^* \iff \alpha p + (1-\alpha)p' \succsim \alpha p^* + (1-\alpha)p'.$$

Expected utility representation of the ranking over income distributions

The ranking over income distributions will be represented using the vNM-expected utility representation. That is,

$$p(y) \sim \int_0^{\bar{y}} U(y)p(y)dy \equiv W^O.$$

Formally, note the slight modification of the vNM-EU theorem in this setting of ranking income distributions:

Theorem 2. *$\succsim \subseteq L \times L$ satisfies $(V1)$ **weak order**, $(V2)$ **continuity**, $(V3)$ **independence** if and only if there exists $U : Y \to \mathbb{R}$ such that, for every $p(y), p^*(y) \in L$,*

$$p(y) \succsim p^*(y) \iff \int_0^{\bar{y}} U(y)p(y)dy \geq \int_0^{\bar{y}} U(y)p^*(y)dy.$$

3.6.1 Ratio measure D

Recall the total social welfare under the two choice models: choice under objective uncertainty and choice under ambiguity.

$$W^O \equiv \int_0^{\bar{y}} U(y)\, p(y)\, dy$$

$$W^A \equiv \min_{\lambda \in C} \int_S \mathbb{E}_{p_s}[U]\, d\lambda.$$

Then the ratio measures are

$$D^O = \frac{W^O}{U(\mu)}, \qquad D^A = \frac{W^A}{U(\mu)}.$$

3.6.2 Equally distributed equivalent measure

Define the EDE wealth levels by

$$U\big(w^O_{EDE}\big) = W^O = \int_0^{\bar{y}} U(y)\,p(y)\,dy, \qquad U\big(w^A_{EDE}\big) = W^A = \min_{\lambda\in C}\int_S \mathbb{E}_{p_s}[U]\,d\lambda.$$

Hence,

$$w^O_{EDE} = U^{-1}(W^O), \qquad w^A_{EDE} = U^{-1}(W^A).$$

The corresponding inequality measures are

$$I^O = 1 - \frac{w^O_{EDE}}{\mu}, \qquad I^A = 1 - \frac{w^A_{EDE}}{\mu}.$$

3.6.3 Closed form under homothetic preferences

For

$$U(y) = \begin{cases} \alpha + \beta \dfrac{y^{1-\epsilon}}{1-\epsilon}, & \epsilon \neq 1, \\ \alpha + \beta \ln y, & \epsilon = 1, \end{cases} \qquad \beta > 0,$$

we obtain the objective and ambiguity formulas by substituting the corresponding welfare aggregator.

If $\epsilon \neq 1$:

$$w^O_{EDE} = \left[\frac{1-\epsilon}{\beta}\big(W^O - \alpha\big)\right]^{\frac{1}{1-\epsilon}}, \qquad w^A_{EDE} = \left[\frac{1-\epsilon}{\beta}\big(W^A - \alpha\big)\right]^{\frac{1}{1-\epsilon}},$$

$$I^O = 1 - \frac{1}{\mu}\left[\frac{1-\epsilon}{\beta}\big(W^O - \alpha\big)\right]^{\frac{1}{1-\epsilon}}, \qquad I^A = 1 - \frac{1}{\mu}\left[\frac{1-\epsilon}{\beta}\big(W^A - \alpha\big)\right]^{\frac{1}{1-\epsilon}}.$$

If $\epsilon = 1$:

$$w^O_{EDE} = \exp\!\left(\frac{W^O - \alpha}{\beta}\right), \qquad w^A_{EDE} = \exp\!\left(\frac{W^A - \alpha}{\beta}\right),$$

$$I^O = 1 - \frac{1}{\mu}\exp\!\left(\frac{W^O - \alpha}{\beta}\right), \qquad I^A = 1 - \frac{1}{\mu}\exp\!\left(\frac{W^A - \alpha}{\beta}\right).$$

So the extension from objective uncertainty to ambiguity is algebraically the replacement

$$W^O \longrightarrow W^A,$$

with the EDE inversion and inequality-index algebra unchanged.

A.4 Median Wealth Gap for the Restricted Sample

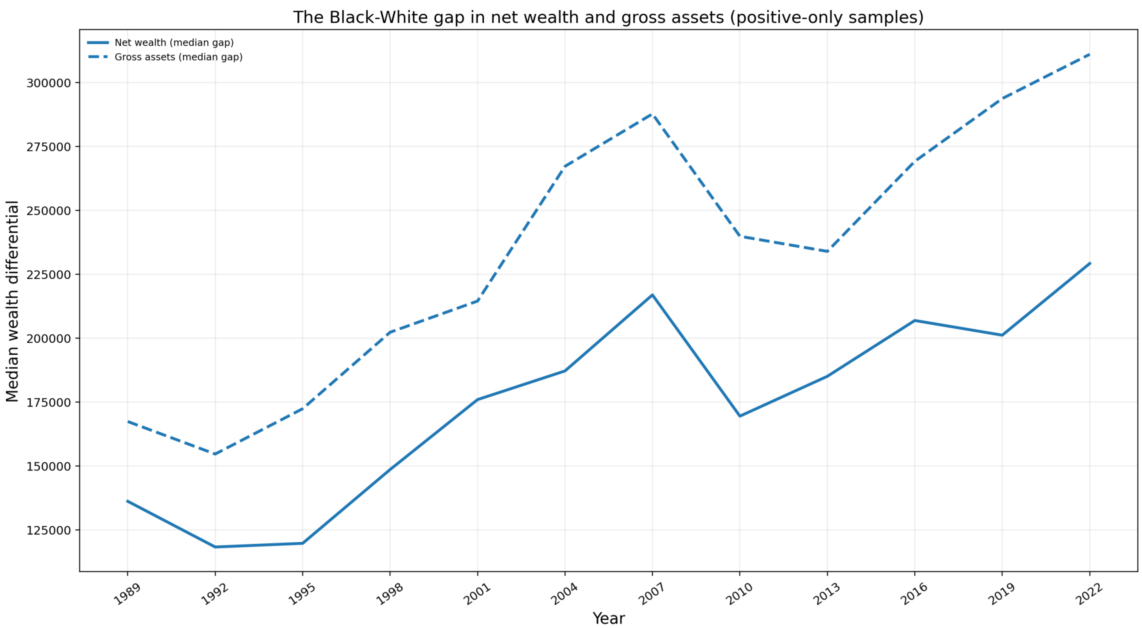

Figure A.1: Black-White median wealth differential in net wealth and gross assets, positive-only samples (SCF 1989–2022).

Even after imposing positive-only restrictions, Figure A.1 shows that both median-gap series still display visible time variation and an upward tendency over the sample. This provides a useful diagnostic benchmark for interpreting the unusually stable, high restricted-sample model-based inequality indices in the main text.

A.5 Diagnostics for Handling Non-positive Wealth

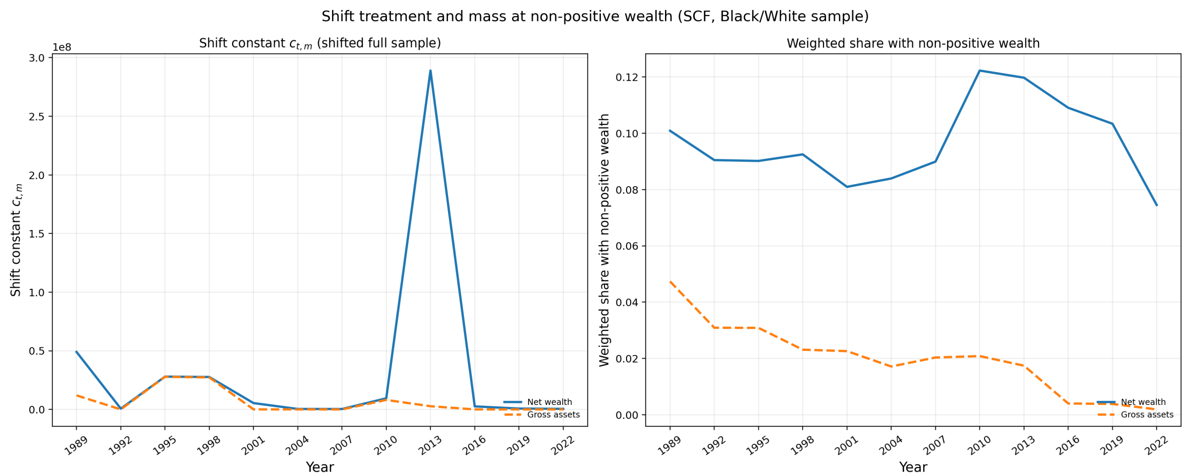

Figure A.2: Left: year-specific shift $c_{t,m}$ applied in the shifted full-sample treatment. Right: weighted fraction of households with non-positive raw wealth for each measure (before shifting).

Together with the median-gap figures, these panels are meant as a simple check on how the sample is affected when households with non-positive wealth are kept in the shifted full-sample treatment (by adding a year-specific constant so utility is well-defined) rather than dropped as in the positive-only restriction. The left panel shows how large that shift had to be each year; the biggest values line up with 2010–2016, which is the period when balance-sheet stress and indebtedness were most salient in the aggregate SCF. The same window shows up in Figure 3.1 as an episode where mean gross assets are comparatively soft in levels while mean net wealth is also compressed—the configuration you expect when liabilities are weighing heavily on net positions. The right panel shows that the weighted share of observations at or below zero in raw wealth is largest in those years as well, for both net wealth and gross assets, so the mechanical "mass at zero or below" lines up with when the shift has to do the most work.

A.6 The Linear Case ($\epsilon = 0$)

When $\epsilon = 0$, the CRRA class collapses to linear utility, $U(y) = \alpha + \beta y$ with $\beta > 0$. Under this case, the objective index satisfies

$$I^O = 0$$

by construction, since the equally distributed equivalent equals the mean. This is the expected no-inequality-aversion benchmark.

In contrast, the ambiguity index remains sensitive to cross-state asymmetry because maxmin aggregation selects the lower state-level expected utility. In the SCF computations, this yields positive I^A values even at $\epsilon = 0$, with notably larger values in the restricted positive-only samples than in most shifted full-sample years.

The $\epsilon = 0$ output is therefore used as a calibration benchmark for the main text: it isolates the linear, no-inequality-aversion case while preserving the ambiguity-vs-objective comparison.